Chapter 15

Wind Energy

15.1 History

The use of wind energy dates back to ancient times when it was employed to propel sailboats. Extensive application of wind turbines seems to have originated in Persia where it was used for grinding wheat. The Arab conquest spread this technology throughout the Islamic world and China.

In Europe, the wind turbine made its appearance in the eleventh century. Two centuries later it had become an important tool, especially in Holland.

The development of the American West was aided by wind-driven pumps and sawmills.

The first significant wind turbine designed specifically for the generation of electricity was built by Charles Brush in Cleveland, Ohio. It operated for 12 years, from 1888 to 1900 supplying the needs of his mansion. Charles Brush was a mining engineer who made a fortune with the installation of arc lights to illuminate cities throughout the United States. His wind turbine was of the then familiar multi-vane type (it sported 144 blades) and, owing to its large solidity (see Section 15.10), rotated rather slowly and required gears and transmission belts to speed up the rotation by a factor of 50 so as to match the specifications of the electric generator.

The wind turbine itself had a diameter of 18.3 meters and its hub was mounted 16.8 meters above ground.

The tower was mounted on a vertical metal pivot so that it could orient itself to face the wind. The whole contraption massed some 40 tons.

Owing to the intermittent nature of the wind, electric energy had to be stored—in this case in 400 storage cells.

Although the wind is free, the investment and maintenance of the plant caused the cost of electricity to be much higher than that produced by steam plants. Consequently, the operation was discontinued in 1900 and from then on the Brush mansion was supplied by the Cleveland utility.

In 1939, construction of a large wind generator was started in Vermont. This was the famous Smith-Putnam machine, erected on a hill called Grandpa's Knob. It was a propeller-type device with a rated power of 1.3 MW at a wind speed of 15 m/s. Rotor diameter was 53 m. The machine started operation in 1941, feeding energy synchronously directly into the power network. Owing to blade failure, in March 1945, operation was discontinued. It ought to be mentioned that the blade failure had been predicted but during World War II there was no opportunity to redesign the propeller hub.

After World War II, the low cost of oil discouraged much of the alternate energy research and wind turbines were no exception. The 1973 oil crises re-spurred interest in wind power as attested by the rapid growth in federal funding. This led to the establishment of **wind farms** that were more successful in generating tax incentives than electric energy. Early machines used in such farms proved disappointing in performance and expensive to maintain. Nevertheless, the experience accumulated led to an approximately 5-fold reduction in the cost of wind-generated electricity. In the beginning of 1980, the cost of 1 kWh was around 25 cents; in 1996 it was, in some installations, down to 5 cents. To be sure, the determination of energy costs is, at best, an unreliable art. Depending on the assumptions made and the accounting models used, the costs may vary considerably. The calculated cost of the kWh depends on an number of factors including

1. The cost of investment—that is, the cost of the installed kW. This number was around \$1000/kW in 1997, and does not appear to have changed much since. The largest windpower plant outside the USA is the 50 MW plant (84 turbines each with a 600 kW capacity) that the French company, Cabinet Germa, installed in Dakhla on the Atlantic coast of Morocco. The project, that started operations in 2000, cost \$60 million, or \$1200/kW.

   These investment costs are comparable with those of fossil-fueled and hydroelectric plants. However, the latter types of plant operate with utilization factors of at least 50% whereas wind power plants operate with a factor of some 20%. The utilization factor compares the amount of energy produced over, say, a year with that that would be produced if the plant operated at full power 24 hours per day. The intermittent and variable nature of the wind is the cause of the low factors achieved. Thus, for a one-on-one comparison, the cost of wind power plants should be multiplied by $0.5/0.2 = 2.5$.

   The one time investment cost must be translated to yearly costs by including the cost of borrowing the necessary funds. See Section 1.12 in Chapter 1. The cost of the kWh produced is extremely sensitive to the cost of the money (which may include interest, taxes, insurance, etc.). This is illustrated in Problem 15.17 at the end of this chapter.
2. Fuel costs, which are, of course, zero for wind and hydroelectric plants.
3. Operating and maintenance costs.
4. Decommissioning costs.
5. Land costs.

Even though the real cost of wind generated energy may be uncertain, what is certain is that it has come down dramatically these last 15 years. In 1997, the selling of wind-generated electricity under a scheme called "green pricing" started becoming popular. In such a scheme, the consumers commit themselves to buying electricity for at least one year in monthly blocks of 100 kWh at typically 2.5 cents/kWh *more* than the going rate. Ecologically minded consumers can thus volunteer to support nonpolluting energy sources.

The relative cheapness of oil in the 1990s resulted in another ebbing in development funds. Nevertheless, the installed wind generating capacity in the USA in January 2004 exceeded 6.3 GW (over 2 GW, in California). The installed capacity in Europe is larger than the one in North America. It had passed the 20 GW mark in January 2003. It should be pointed out that the installed wind turbine capacity in the US corresponded (2004) to only some 0.6% of the total electricity generating capacity.

The energy produced by wind farms is high-utility electricity whereas the energy from fuel is low-utility heat. Thus, to be fair, wind energy should be multiplied by 3 when comparison is made with fossil fuel use to reflect the roughly 30% conversion efficiency from the latter to electricity.

Notwithstanding the small present day contribution of wind energy to the total energy picture, there is merit in pursuing aggressively the technology, especially in view of its favorable ecological aspects:

1. Clearly, wind power emits absolutely no CO_2, by far the major pollutant when fuels (other than hydrogen or biomass) are burned.
2. The operation of wind turbines leaves behind no dangerous residues as do nuclear plants.
3. Decommissioning costs of wind turbines are much smaller than those of many other types of power plants especially compared with those of nuclear generators.
4. Land occupied by wind farms can find other uses such as agriculture.

Some groups are opposed to wind turbines because of the danger they constitute to the birds that fly near the wind farms.

The optimal size of a wind turbine has been the subject of disputes these last few decades. Government sponsored research in both the USA and Germany favored large (several MW) machines, while private developers opted for much smaller ones. Large machines fitted in well with the ingrained habits of the power generation industry accustomed to the advantages of economy of scale. Such advantages, however, do not seem to apply to wind turbines. Consider an extremely oversimplified reasoning:

For a given wind regimen, the amount of energy that can be abstracted from the wind is proportional to the swept area of the turbine. The area swept out by a rotor with 100 m diameter is the same as that of 100 machines with 10 m diameter. The mass of the plant (in a first-order scaling) varies with the *cube* of the diameter. The aggregate mass of the 100 small machines is only 10% of the mass of the large one. Hence, for the same amount of energy produced, the total equipment mass varies inversely with the diameter. Since costs tend to grow with mass, many small turbines ought to be more economical than one large one. This reasoning would suggest that the best solution is to use an infinite number of infinitely small turbines. Taken to this extreme, the conclusion is patently absurd.

Other factors play an important role in the economy (or diseconomy) of scale for wind turbines, complicating the situation to the point that a plausible model of how energy cost varies with turbine size becomes difficult to construct. Larger machines could arguably be more efficient and might simplify maintenance. They also would require less ancillary equipment (for instance, one single large transformer instead of many small ones) and possible less land area. Small machines profit from mass production economies, from modularity (allowing an easy expansion of the capacity of a wind farm) and from a greater immunity to breakdown (the break-down of a few turbines affects only a fraction of the total wind farm capacity).

One of the largest early wind turbines tested was the German "Growian"† (100-m diameter, 3 MW) and the Boeing Mod-5B (98-m diameter, 3.2 MW).

The development of large wind machines seems to have been temporarily abandoned. On the other hand, the size of the turbines in large practical wind farms has been growing. In 1996, sizes between 500 kW and 750 kW were being favored; larger ones are now being installed. Thus, the trend toward larger turbines continues.

15.2 Wind Turbine Configurations

Several wind turbine configurations have been proposed, including:

1. drag-type,
2. lift-type (with vertical or horizontal axes),
3. Magnus effect wind plants,
4. Vortex wind plants.

Essentially all present day wind turbines are of the lift type and, over 90% of these are of the horizontal axis type. Magnus effect and vortex plants have never played a serious practical role.

15.2.1 Drag-Type Wind Turbines

In a drag-type turbine, the wind exerts a force in the direction it is blowing—that is, it simply pushes on a surface as it does in a sailboat sailing before the wind. Clearly, the surface on which the wind impinges cannot move faster than the wind itself.

† The German love for acronyms has given us words like *Stuka* and *Flak*. *Growian* stands for "Grosse Wind Energie Anlage."

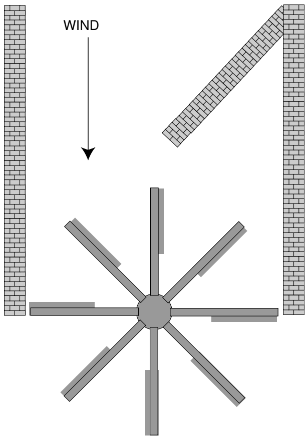


Figure 15.1 Top view of an ancient Persian wind turbine.

The ancient Persian wind turbine was a drag-type machine. Figure 15.1 is a sketch of such a mill seen from above. It consisted of a vertical axis to which horizontal radial arms were attached. Near the extremities of these arms, a vertical curtain was installed and this was the surface on which the wind exerted its useful force. Two walls channeled the wind, forcing it to blow on only one side of the device, thus creating a torque. Notice that one wall forms a funnel concentrating the collected wind.

The bucket wind turbine, sketched in Figure 15.2, is another vertical-axis drag-type device. It rotates because the convex surface offers less wind drag than the concave one. This device can be cheaply built by amateurs using an oil barrel cut along its vertical axis. It operates inefficiently.

Figure 15.2 A 2-bucket wind turbine.

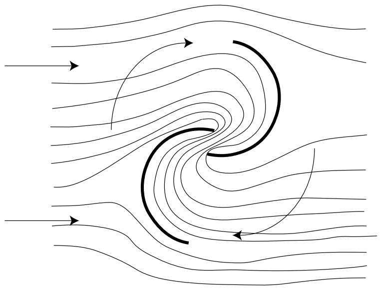


Figure 15.3 Air flow in a Savonius rotor.

15.5

Improved performance can be obtained by staggering the buckets as shown in Figure 15.3 so that a gap is left between them. The air is accelerated as it passes the gap reducing the front drag of the convex bucket. It is then blown on the reverse side of the bucket aiding in the creation of torque. This type of device is called a **Savonius** rotor and actually uses a certain amount of lift (in addition) to drag.

Savonius turbines cannot compete in efficiency with pure lift-type machines, but they are easy to build and find application as sensors in anemometers and eolergometers and as starters for vertical-axis lift-type machines.

15.2.2 Lift-Type Wind Turbines

In a lift-type machine, the wind generates a force perpendicular to the direction it is blowing. The familiar propeller wind turbines are of the horizontal-axis, lift-type. All lift-type turbines are analogous to sailboats sailing cross wind. The sailboat (or the blade of the turbine) can move substantially faster than the wind itself. Figure 15.4 shows such turbines.

Notice that the propeller-driven shaft that delivers the collected energy is high above ground level. This usually forces one of two solutions: either the electric generator is placed on top of the tower next to the propeller, or a long shaft, with associated gears, is used to bring the power to a ground-level generator. The first solution, although requiring reinforced towers, is the preferred one because of the cost and difficulties of transmitting large mechanical power over long shafts. Mounting the generator on top of the tower increases the mass of that part of the system that has to swivel around when the wind changes direction.

Some wind turbines have the propeller upstream from the generator and some downstream. It has been found that the upstream placement reduces the noise produced by the machine.

A propeller wind turbine that employs a ground-level generator but avoids the use of a long shaft is the suction-type wind turbine. It resembles a conventional wind turbine but the rotating blades act as a centrifugal pump. The blades are hollow and have a perforation at the tip so that air is expelled by centrifugal action creating a partial vacuum near the hub. A long pipe connects the hub to an auxiliary turbine located at ground level. The inrushing air drives this turbine. The system does not seem promising enough to justify further development.

One wind turbine configuration not only allows placing the generator on the ground but also avoids the necessity of reorienting the machine every time the wind changes direction—it is the vertical-axis lift-type wind turbine. The one illustrated in the center of Figure 15.4 is a design that was proposed by McDonnell-Douglas and was called "Gyromill." It would have been capable of generating 120 kW, but it was never commercialized.

15.6

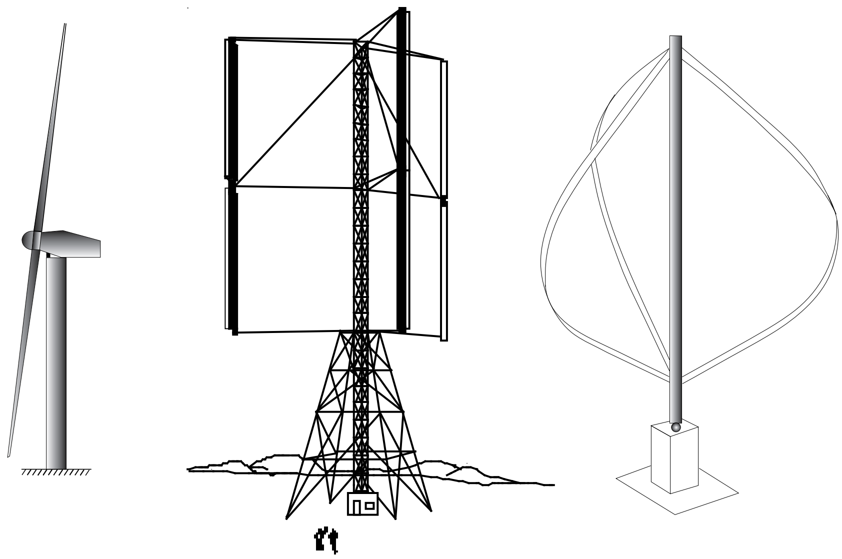


Figure 15.4 From left to right: a horizontal axis (propeller) type turbine, and two vertical axis machines—a "Gyromill" and a Darrieus.

One obvious disadvantage of the gyromill is the centrifugal force that causes the wings to bend outward, placing considerable stress on them. An elegant way to avoid centrifugal stresses is to form the wings in the shape assumed by a rotating rope loosely attached to the top and bottom of the rotating shaft. This leads to the familiar "egg beater" shape and, of course, causes the wing to work only in tension.

The shape of such a rotating rope is called a **troposkein** and resembles closely a **catenary**. There is, however, a difference. The catenary is "the shape assumed by a perfectly flexible inextensible cord of uniform density and cross-section hanging freely from two fixed points." Each unit length of the cord is subject to the same (gravitational) force. In the case of the troposkein, the force acting on each section of the cord depends on the distance of the section from the axis of rotation.

The troposkein wing (right-hand drawing of Figure 15.4) was first suggested by a French engineer called Darrieus after whom this type of wind turbine is named.

15.2.3 Magnus Effect Wind Machines

Magnus effect machines have been proposed but look unpromising. This effect, discussed in Section 15.14, is the one responsible for, among other things, the "curve" in baseball.

When a pitcher throws a curve, he causes the ball to spin creating an asymmetry: one side of the ball moves faster with respect to the air than the other and, consequently, generates the "lift" that modifies the trajectory of the ball. An identical effect occurs when a vertical spinning cylinder is exposed to the wind. The resulting force, normal to the wind direction, has been employed to move sailboats and wind machines.

15.2.4 Vortex Wind Machines

Finally, it is possible to abstract energy from the wind by making it enter tangentially through a vertical slit into a vertical hollow cylinder. As a result, the air inside is forced to gyrate and the resulting centrifugal force causes a radial pressure gradient to appear. The center of this air column, being at lower than atmospheric pressure, sucks outside air through openings at the bottom of the cylinder. The inrushing air drives a turbine coupled to a generator. The spinning air exits through the open top of the cylinder forming a vortex continuously swept away by the wind. This type of machine has been proposed by Gruman.

15.3 Eolergometry

Later on in this chapter, we are going to show that the power of a wind turbine is proportional to the cube of the wind velocity:

$$P_D = \frac{16}{27}\tfrac{1}{2}\rho v^3 A\eta \tag{1}$$

where $\frac{1}{2}\rho v^3$ is the **power density** in the wind, $\frac{16}{27}\frac{1}{2}\rho v^3$ is the **available power density** from the wind, A is the **swept area** of the wind turbine and η is the **efficiency** of the wind turbine.

The mean power output from the wind turbine over a period from 0 to T is proportional to the cube of the **mean cubic wind velocity**, $<v>$:

$$<v> \equiv \left(\frac{1}{T}\int_0^T v^3 dt\right)^{1/3}. \tag{2}$$

Owing to this cubic dependence on velocity, local wind conditions exert a strong influence on the useful energy generated by wind turbines, and site selection plays a critical role in overall planning.

Anemometers—instruments that measure or record wind velocity—can be used in wind surveys. Anemometric records have to be converted to eolergometric data—that is, data on wind power density. The mean cubic velocity, $<v>$, must be calculated from velocity measurements taken at frequent intervals.

The usual anemometric averages, $\overline{v}$—that is, the arithmetical average of v, are not particularly suitable for siting wind turbines. Consider a wind that blows constantly at a speed of 10 m/s (average speed of $\overline{v} = 10$ m/s). It carries an amount of energy proportional to $v^3 = 1000$. A wind that blows at 50 m/s 20% of the time and remains calm the rest of the time also has a $\overline{v}$ of 10 m/s, yet the energy it carries is proportional to $0.2 \times 50^3 = 25,000$ or 25 times more than in the previous case. In the first case, $<v>= 10$ m/s, while in the second, it was 29.2 m/s.

The quantity, $<v>$, is not normally measured directly by meteorologists although it can, as stated above, be derived from anemograms. Instruments that measure the cubic mean velocity, or else the total available energy density over a prescribed period of time are called **eolergometers** from *Aeolus* (god of winds) + *ergon* (work) + *metrein* (to measure). Such instruments are useful in surveys of the availability of wind energy.

Eolergometric surveys are complicated by the variability of the wind energy density from point to point (as a function of local topography) and by the necessity of obtaining vertical wind energy profiles. It is important that surveys be conducted over a long period of time—one year at least—so as to collect information on the seasonal behavior.

Clearly, values of $\overline{v}$ are easier to obtain than those of $<v>$ and, consequently, there is the temptation to guess the latter from the former. However, the ratio $\gamma \equiv <v> / \overline{v}$ is a function of the temporal statistics of the wind velocity and is strongly site dependent. For perfectly steady winds, it would, of course, be 1. For the extreme case of the example in one of the preceding paragraphs, it is 2.92. Since the wind power density is proportional to the cube of the velocity, uncertainties in this ratio lead to large errors in the estimated available energy. Nevertheless, it has been suggested that a ratio of 1.24 can be used to make a (wild) guess of the energy availability at California sites.

When no information on the variation of wind velocity with height above ground is available, one can use the scaling formula

$$v(h) = v(h_0)\left(\frac{h}{h_0}\right)^{1/7}. \tag{3}$$

15.4 Availability of Wind Energy

In Chapter 1, we saw that 30% of the 173,000 TW of solar radiation incident on Earth is reflected back into space as the planetary albedo. Of the 121,000 TW that reach the surface, 3% (3,600 TW) are converted into wind energy and 35% of this is dissipated in the lower 1 km of the atmosphere. This corresponds to 1200 TW. Since humanity at present uses only some 7 TW, it would appear that wind energy alone would be ample to satisfy all of our needs.

This kind of estimate can lead to extremely overoptimistic expectations. For one thing, it is difficult to imagine wind turbines covering all the ocean expanses. If we restrict wind turbines to the total of land areas, we would be talking about 400 TW. Again, it would be impossible to cover all the land area. Say that we would be willing to go as far as 10% of it, which is more than the percentage of land area dedicated to agriculture. We are now down to 40 TW. But, owing to the cubic dependence of power on wind speed, it is easy to see that much of this wind energy is associated with destructive hurricane-like winds, which actually generate no energy, since any reasonable wind turbine must shut itself down under such conditions.

The difficulty is that wind energy is very dilute. At a 10 m height, the wind power density may be some 300 W/m^2 at good sites and, at 50 m height, it can reach some 700 W/m^2, as it does in the San Gregorio Pass in California and in Livingston, Montana. Notice that these are not values of *available* wind power densities, which are 59% (16/27) of the preceding values. See Subsection 15.6.5.

Any sizable plant requires a large collecting area which means many turbines spread over a large area of land.†

Winds tend to be extremely variable so that wind plants must be associated with energy storage facilities (usually in small individual installations) or must debit into the power distribution network (in large wind farms).

In some areas of the world where the wind is quite constant in both speed and direction, wind turbines would operate with greater efficiency. All along the northeastern coast of Brazil, trade winds blow almost uninterruptedly from a northeasterly direction with steady speeds of some 13 knots (about 7 m/s) which corresponds to 220 W/m^2 of wind power density or 130 W/m^2 of *available* power density. This constant wind direction would allow the construction of fixed wind concentrators capable of substantially increasing the wind capture area.

15.5 Wind Turbine Characteristics

To minimize the cost of the produced electricity, the rated power of the generator must bear an appropriate relationship to the swept area of the wind turbine. The rated power of a generator is the maximum power it can deliver under steady conditions. Usually, the power actually generated is substantially lower than the rated. The ratio of the rated generator power to the swept area of the turbine is called **specific rated capacity** or **rotor loading**. Modern machines use rotor loadings of 300 to 500 W/m^2.

The ideal wind turbine would be tailored to the wind conditions of each

† Of course, land used for wind farms may be still available for agriculture.

site. With steady unvarying winds ($\gamma = 1$) the rated power of the generator should be the same as that of the turbine. However, when $\gamma > 1$, which is invariably the case, the most economical combination has to be determined by considering many different factors. See, for example, Problem 15.3.

The Boeing Model 2 wind turbine was of the horizontal-axis (propeller) type. The rotor had a diameter of 91.5 m with variable pitch in the outer 14 m of each blade. This allowed the control of start-up and shutdown as well as the adjustment of its rotating speed. The performance of this wind turbine is shown in Figure 15.5. Note that with wind speeds below 3.9 m/s, the propeller does not rotate; at 6.3 m/s the machine reaches 17.5 rpm and its output is synchronized with the power grid.

The power generated increases rapidly up to wind velocities of 12.5 m/s and then remains constant up to 26.8 m/s.

Above this speed, the machine shuts itself down for safety reasons. It can, however, withstand winds up to some 56 m/s. With this arrangement, at high wind speeds, the turbine extracts but a small fraction of the available energy. The Model 2 generated 2.5 MW of electricity with any wind in the 12.5 m/s to 25 m/s range notwithstanding there being 8 times more energy at the higher speed. At even higher wind speeds, the machine shuts itself down to avoid destructive stresses, and, thus, delivers no energy just when there is a largest amount of power in the wind.

Typically, the power delivered by a given wind plant depends on the wind velocity in a manner similar to the one displayed in Figure 15.5.

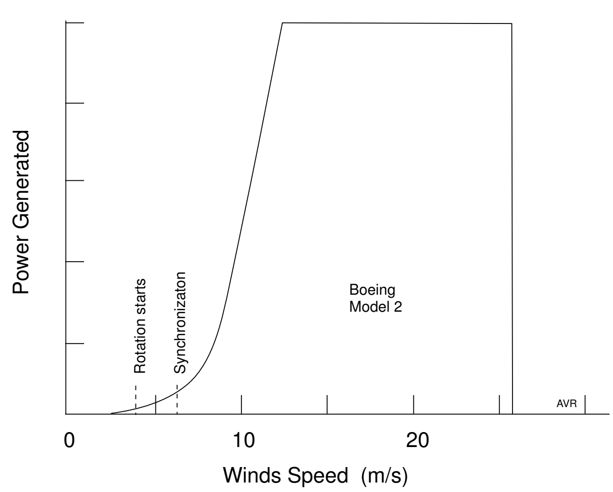


Figure 15.5 Power output of the Boeing Model 2 wind turbine as a function of wind velocity.

Wind turbines frequently deliver their energy to a utility-operated net and must do so with alternating current of the correct frequency. There are two general solutions to the synchronization problem:

1. Maintaining the rotation of the turbine at a constant rate (by changing the blade pitch, for instance).
2. Allowing the turbine to rotate at the speed dictated by load and wind velocity. In this case dc is generated and electronically "inverted" to ac. Such **variable-speed** machines are somewhat more expensive but are more efficient and have a longer life.

15.6 Principles of Aerodynamics

The symbol, P, in this chapter stands for both *power* and *power density*—that is, power per unit area, depending on the context. The lower case, p, is reserved for *pressure*.

The following subscripts are used:

$P_W = \frac{1}{2}\rho v^3$	"Power density in the wind." This is the amount of energy transported across a unit area in unit time.
$P_A = \frac{16}{27}\frac{1}{2}\rho v^3$	"Available Power density." This is the theoretical maximum amount of power that can be extracted from the wind.
$P_D = \frac{16}{27}\frac{1}{2}\rho v^3 A\eta$	"Power delivered." This is the power that a wind turbine delivers to its load.

15.6.1 Flux

The flux of a fluid is defined as the number of molecules that cross a unit area (normal to the flow) in unit time. It can be seen that, if n is the concentration of the molecules (number per unit volume) and v is the bulk velocity of the flow, then the flux, ϕ is

$$\phi = nv. \qquad \mathrm{m^{-2}s^{-1}} \qquad (4)$$

The total flow across an area, A, is, consequently

$$\Phi = \phi A. \qquad \mathrm{s^{-1}} \qquad (5)$$

15.6.2 Power in the Wind

If the mean mass of the gas molecules is m, then the mean energy of a molecule owing to its bulk drift (not owing to its thermal motion) is $\frac{1}{2}mv^2$. The amount of energy being transported across a unit area in unit time is the power density of the wind:

$$P_W = \tfrac{1}{2}mv^2\phi = \tfrac{1}{2}mnv^3 = \tfrac{1}{2}\rho v^3. \qquad \text{W m}^{-2} \qquad (6)$$

Notice that the power density is proportional to the cube of the wind velocity. The quantity, ρ, is the gas density—that is, the mass per unit volume:

$$\rho = mn. \qquad \text{kg m}^{-3} \qquad (7)$$

At standard temperature and pressure (STP),† the density of air is

$$\rho = \frac{0.2 \times 32 + 0.8 \times 28}{22.4} = 1.29. \qquad \text{kg m}^{-3} \qquad (8)$$

The numerator is the average molecular mass of air containing 20% O_2 and 80% N_2, by volume. The denominator is the number of cubic meters per kilomole at STP:

From the perfect gas law, at STP,

$$V = \frac{RT}{p} = \frac{8314 \times 273.3}{1.013 \times 10^5} = 22.4 \quad \text{m}^3.$$

15.6.3 Dynamic Pressure

Since 1 m^3 of gas contains n molecules and each molecule carries $\frac{1}{2}mv^2$ joules of energy owing to its bulk motion, the total energy density—that is, the total energy per unit volume is

$$W_d = \tfrac{1}{2}nmv^2 = \tfrac{1}{2}\rho v^2. \qquad \text{J m}^{-3} \text{ or N m}^{-2} \qquad (9)$$

Energy per unit volume has the dimensions of force per unit area—that is, of pressure. Thus W_d is referred to as **dynamic pressure**.

15.6.4 Wind Pressure

Wind exerts pressure on any surface exposed to it. Consider the (unrealistic) flow pattern depicted in Figure 15.6. The assumption is that any

† STP corresponds to one atmosphere and 0 celsius.

molecule striking the surface is reflected and moves back against the wind without interfering with the incoming molecules.

Under such a simplistic assumption, each molecule transfers to the surface a momentum, $2mv$, because its velocity change is $2v$ (it impacted with a velocity, v, and was reflected with a velocity, $-v$). Since the flux is nv, the rate of momentum transfer per unit area, i.e., the generated pressure, is $2mv \times nv = 2\rho v^2$. The assumption is valid only at very low gas concentrations, when indeed, a molecule bouncing back may miss the incoming ones.

In a more realistic flow, the reflected molecules will disturb the incoming flow which would then roughly resemble the pattern shown in Figure 15.7. This leads to a pressure smaller than that from the ideal flow case, a pressure that depends on the shape of the object. To treat this complicated problem, aerodynamicists assume that the real pressure is equal to the dynamic pressure multiplied by an experimentally determined correction factor, C_D, called the **drag coefficient**.

$$p = \frac{1}{2}\rho v^2 C_D. \qquad (10)$$

The drag coefficient depends on the shape of the object and, to a certain extent, on its size and on the flow velocity. This means, of course, that the pressure exerted by the wind on a surface is not strictly proportional to v^2 as suggested by Equation 10.

The drag coefficient of a large flat plate at low subsonic velocities is usually taken as $C_D = 1.28$.

15.6.5 Available Power

Electrical engineers are familiar with the concept of available power. If a source (see Figure 15.8) has an open circuit voltage, V, and an internal resistance, R_s, the maximum power it can deliver to a load is $V^2/4R_s$. This occurs when $R_L = R_s$.

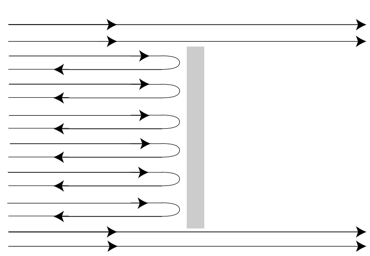


Figure 15.6 A simplistic flow pattern.

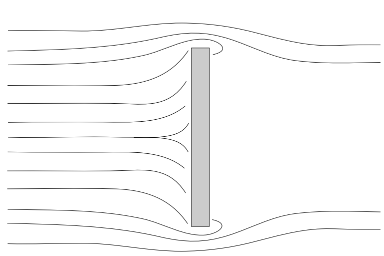


Figure 15.7 A more realistic flow pattern.

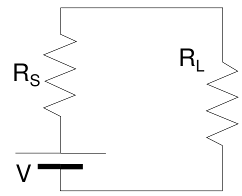


Figure 15.8 An electric source and its load.

The same question arises when power is to be extracted from the wind. If the surface that interacts with the wind is stationary, it extracts no power because there is no motion. If the surface is allowed to drift downwind without any resistance, then it again will extract no power because the wind will exert no force on it. Clearly, there must be a velocity such that maximum power is extracted from the wind.

The power density, P, extracted from the wind is the product of the pressure, p, on the surface and the velocity, w, with which the surface drifts downwind. The wind pressure is

$$p = \frac{1}{2}\rho C_D (v - w)^2, \tag{11}$$

hence

$$P = pw = \frac{1}{2}\rho C_D (v - w)^2 w. \tag{12}$$

Setting $\partial P/\partial w$ to zero, an extremum of P is found. This is a maximum and occurs for $w = v/3$ independently of the value of C_D. Thus,

$$P_{max} = \frac{2}{27}\rho C_D v^3. \qquad \text{W m}^{-2} \tag{13}$$

The ratio of maximum extractable power to power in the wind is

$$\frac{P_{max}}{P_w} = \frac{\frac{2}{27}\rho C_D v^3}{\frac{1}{2}\rho v^3} = \frac{4}{27} C_D. \tag{14}$$

The largest possible value of C_D is that predicted by our simplistic formula that, by stating that $p = 2\rho v^2$, implies that $C_D = 4$. Thus, at best, it is possible to extract 16/27 or 59.3% of the "power in the wind." This is the **available power density** from the wind:

$$P_A = \frac{16}{27}\frac{1}{2}\rho v^3. \tag{15}$$

15.6.6 *Efficiency of a Wind Turbine*

The efficiency of a wind turbine is the ratio of the power, P_D, delivered to the load, to some reference power. There is a certain amount of arbitrariness in this definition. Some authors choose the "power in the wind" ($\frac{1}{2}\rho v^3$) as reference. In this text we will use the available power (P_A):

$$\eta = \frac{P_D}{P_A}. \tag{16}$$

In a well-designed wind turbine, the efficiency can reach 0.7.

15.15

15.7 Airfoils

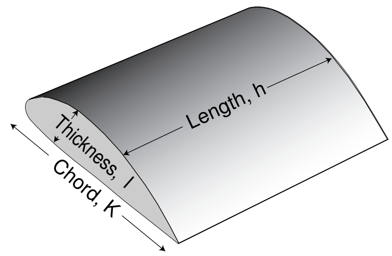


Figure 15.9 Significant dimensions in an airfoil.

Airplane wings, helicopter rotors, empennage surfaces, and propeller blades are examples of aerodynamic surfaces (**airfoils**), which must generate a great deal of lift with a correspondingly small drag. It is intuitive that the performance of an airfoil depends greatly on the shape of its cross section.

Figure 15.10 shows a section through an airfoil. The line (A.A') represents the trace of an arbitrary **reference plane.** Notice that the region above this plane differs from the one below it—the airfoil is **asymmetric**. In **symmetric** airfoils, the reference plane is the plane of symmetry, and the region above it is a mirror image of that below.

When air flows relative to the airfoil along the x-axis in the figure, a force is exerted on the foil. Such force is usually decomposed into a **lift** component (normal to the velocity) and a **drag** component (parallel to the velocity). The corresponding pressures are indicated by the vectors, p_L and p_D, in the figure. The angle between the wind direction and the reference line is called the **angle of attack**, α.

For each shape of the airfoil, p_L and p_D are determined experimentally in wind tunnels under specified conditions. The observed pressures are related to the dynamic pressure, $\frac{1}{2}\rho v^2$, by proportionality constants, C_L and C_D, called, respectively, the **lift coefficient** and the **drag coefficient**:

$$p_L = \frac{1}{2}\rho v^2 C_L, \tag{17}$$

$$p_D = \frac{1}{2}\rho v^2 C_D. \tag{18}$$

These coefficients are function of the angle of attack† and can be found in tabulations, many of which were prepared by NACA (the forerunner of NASA) in the United States and by Göttingen in Germany. Since, presumably, airplanes move only forward, the tabulations are usually made for only a small range of angles of attack near zero. However, for some airfoils, data are available for all 360° of α as in Figure 15.11 which shows the dependence of C_L on α for the airfoil known as Göttingen 420.

† The coefficients depend also, although more weakly, on the Reynolds number, R, and the aspect ratio, Λ, as we are going to show.

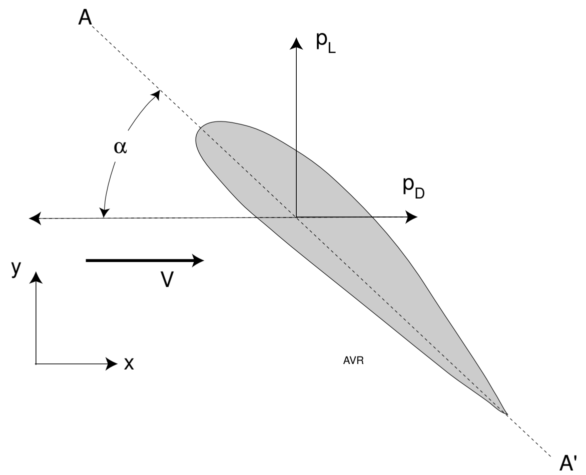


Figure 15.10 Pressure on an airfoil.

Notice that the airfoil under discussion generates lift even with negative angles of attack (as long as they remain small). When there is a positive angle of attack, one can intuitively understand the creation of lift even for a flat surface—after all the airflow is hitting the surface from below and the drag it exerts has a lift component. However, when the angle of attack is zero or slightly negative, the lift must be due to more complicated mechanisms. Observations show that the air pressure immediately above the airfoil is smaller than the pressure immediately below it and this is the obvious cause of the lift. The problem is to explain how such a pressure difference comes about.

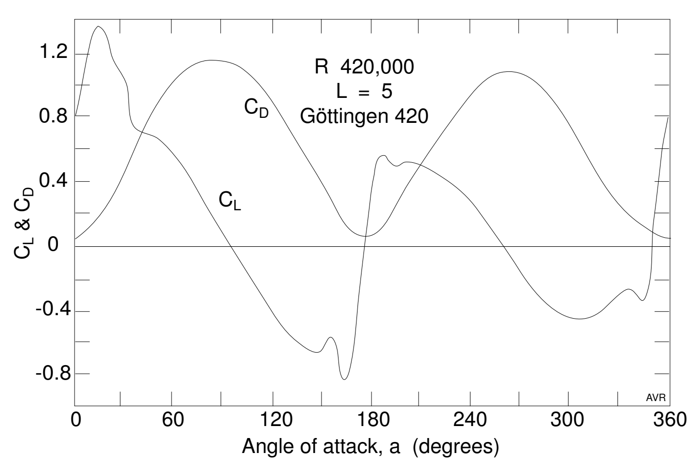


Figure 15.11 The lift and drag coefficients of the Göttingen 420 airfoil.

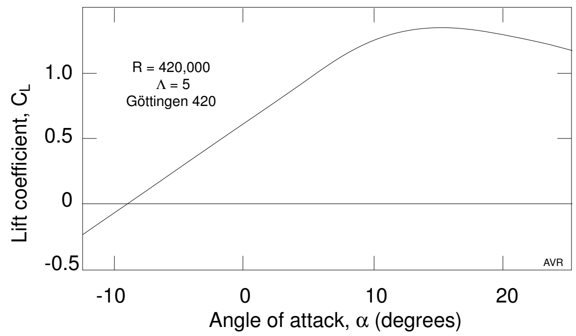


Figure 15.12 The lift coefficient of the Göttingen 420 airfoil.

Further observations show also that

1. the air flow tends to follow the curvature of the top of the airfoil instead of simply being deflected away from it;
2. the air flow velocity is substantially increased lowering the pressure, according to Bernoulli's principle.

We need some explanation for these observations.

The Coanda† Effect is the cause of the air flow's tendency to follow the shape of the airfoil. It is extremely easy to demonstrate this phenomenon. Open a faucet and allow a thin stream of water to fall from it. Now take a curved surface—a common drinking glass will do—and let the stream hit the side of the glass at a glancing angle. The water will run along the side and then, making a sharp turn, will flow along the bottom instead of simply falling vertically down. The water tends to follow the glass surface just as the air tends to follow the airfoil surface.

The bulging part of the airfoil restricts the air flow (as in the strangulation in a venturi) causing an acceleration of the flow. The transit time of air molecules along the path over the airfoil from leading to trailing edge is not the same as that for the flow under the airfoil. Such synchronism, frequently invoked in explanations of wing lift, does, in fact, not occur.

In the airfoil of Figure 15.12, the lift is linearly related to α up to some 10°. At higher angles the airfoil **stalls**—that is, a further increase in α actually reduces the lift. Near zero angle of attack, this airfoil develops a lift over 16 times larger than its drag.

Because the lift and drag coefficients are not strictly independent of the air velocity or the dimensions of the wing, it is impossible to scale any experimental results exactly. However, the data are valid for different sizes and speeds as long as the **Reynolds number** is preserved.

† Henri-Marie Coanda, Romanian scientist (1885–1972) described this effect in 1930.

15.8 Reynolds Number

The size of most airplanes exceeds, by far, that of available wind tunnels causing engineers to do their tests and measurements on reduced scale models. The dimensions of such models are an accurate constant fraction of the original. One thing, however, can usually not be scaled down proportionally: the size of the molecules in air. For this reason, measurement on models may not be converted with precision to expected forces on the real plane. In fact, the forces observed on an object moving in a fluid are not only the **dynamic** forces, F_d (proportional to $\frac{1}{2}\rho v^2$) but include also **viscous** forces, F_v.

When an airfoil moves through still air, molecules of gas in immediate contact with the surface are forced, through friction, to move with the velocity, v_x, (assuming movement in the x-direction) of the foil, while those at a large distance do not move at all. A velocity gradient, ∇v_x, is established in the y-direction.

The resulting velocity shear causes the viscous force, F_v, to appear. This force is proportional to the wing area, A, to the velocity gradient, $\partial v_x/\partial y$, and to the coefficient of viscosity, μ, a property of the medium through which the wing moves.

$$F_v = \mu \frac{\partial v_x}{\partial y} A. \tag{19}$$

For accurate scaling, it is necessary to preserve the ratio of the dynamic to the viscous forces. Define a quantity, R, called **Reynolds number** as

$$R \propto \frac{F_d}{F_v} = \frac{\frac{1}{2}\rho v_x^2 A}{\mu \partial v_x/\partial y A} \propto \frac{\rho}{\mu} . \frac{v_x^2}{\partial v_x/\partial y} \tag{20}$$

This, is too complicated. Make two simplifying assumptions:

1. $\frac{\partial v_x}{\partial y}$ is independent of y, and
2. the air is disturbed only to a distance, K, above the wing (where K is the **chord length**).

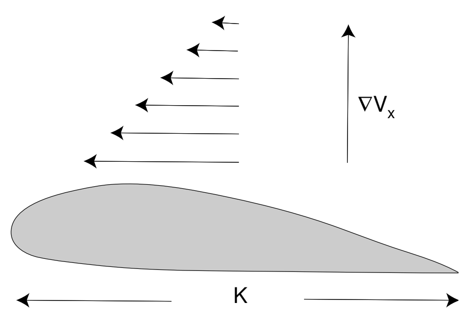


Figure 15.13 Wind shear over a wing.

Under such circumstances,

$$\frac{\partial v_x}{\partial y} = \frac{v}{K}, \tag{21}$$

and

$$R = \frac{\rho v^2}{\mu v/K} = \frac{\rho}{\mu} v K. \tag{22}$$

The value of μ for air is 1.84×10^{-5} kg m^{-1} s^{-1} and is, contrary to one's gut feeling, independent of pressure and density (see the box at the end of this section). However, the ratio, μ/ρ, called the **kinematic viscosity** increases when the pressure decreases. Fluids, at low pressure exhibit great kinematic viscosity and this explains why vacuum pumps need large diameter pipes. For STP conditions, this ratio for air is 1/70,000 m^2/s because $\rho = 1.29$ kg m^{-3}.

Since for any given angle of attack, the coefficients (C_L and C_D) are functions of R, measurements made with models cannot be extrapolated to life-size wings unless the Reynolds number is the same. Fortunately the coefficients depend only weakly on R as illustrated in Figure 15.14, where C_D and $C_{L_{max}}$ are plotted versus R for the NACA 0012 symmetric airfoil. $C_{L_{max}}$ is the largest value that C_L can reach as a function of the angle of attack.

For first order calculations, one can ignore the effects of varying Reynolds number. In more precise calculations, the correct Reynolds number must be used especially because, as we shall see, the wing of a vertical axis wind turbine perceives a variable wind velocity throughout one cycle of its rotation.

In general, things tend to improve with larger Reynolds numbers (because the lift-over-drag ratio usually increases). This means that, on these grounds, larger wind turbines tend to be more efficient than smaller ones.

To measure the characteristics of this wing using a model with a 0.3 m chord under the same Reynolds number, one would need a wind velocity of 3,600 m/s. However, the result would not be valid because the speed in question is supersonic. This explains why much of the wind tunnel data correspond to modest Reynolds numbers. The Göttingen-420 data were measured at $R = 420,000$.

A 3-meter chord wing moving at 360 km/h has a Reynolds number of

$$R = 7 \times 10^4 \times 100 \times 3 = 21 \times 10^6. \tag{23}$$

There is a class of wind tunnels, the NACA **variable-density tunnels**, in which R is increased by increasing the static air pressure. This raises ρ but does not affect μ. Thus, large Reynolds numbers can be achieved with small models at moderate wind speeds.

15.20

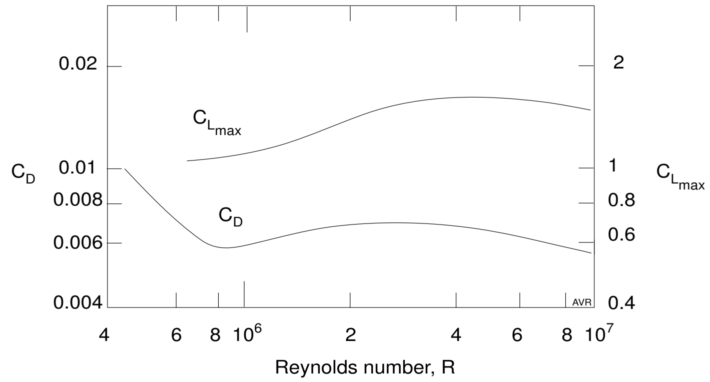


Figure 15.14 Effect of the Reynolds number on the lift and drag coefficients.

Why is μ independent of pressure or density? When a molecule of gas suffers an (isotropic) collision, the next collision will, statistically, occur at a distance one mean free path, ℓ, away. This locates the collision on the surface of a sphere with radius, ℓ, centered on the site of the previous collision. The projected area of this sphere is proportional to ℓ^2. One can, therefore, expect that

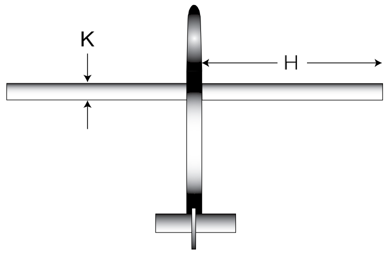
$$\mu \propto \nu n \ell^2. \tag{24}$$

where ν is the **collision frequency** and n is the **concentration** of the molecules. But ℓ is inversely proportional to the concentration while the collision frequency is directly proportional to it. Thus,

$$\mu \propto n \times n \times \frac{1}{n^2}, \tag{25}$$

i.e., μ is independent of n.

15.9 Aspect Ratio



In a rectangular wing, the ratio between the length, H, and the chord, K, is called the **aspect ratio**, $\mathcal{A\!R}$.

$$\mathcal{A\!R} \equiv \frac{H}{K} = \frac{H^2}{KL} = \frac{H^2}{A}, \tag{26}$$

where A is the area of the wing.

Hence, the aspect ratio can be defined as the ratio of the square of the wing length to the wing area. This definition must be used in the case of tapered (nonrectangular) wings, which have a variable chord length.

The drag experienced by a body moving through a fluid is caused by a number of different mechanisms. An ideal infinitely smooth and infinitely long wing would experience only a **pressure drag**. However, real wings are not perfectly smooth and the air tends to adhere to the surface causing viscous shearing forces to appear and generating a **skin-friction drag**.

Wing lift is the result of the pressure being larger under the wing than over it. At the tip, there is a "short circuit" between the under side and the top and air circulates around the tip forming a vortex. Energy is used to impart the circular motion to the air in the vortex and this energy must come from the forward motion of the wing. It manifests itself as an additional drag force called the **induced drag.** The induced drag can lessened by

1. increasing the number of wings (or the number of wing tips),
2. increasing the aspect ratio of the wing,
3. tapering the wing so that the chord is smaller near the tip, and
4. placing a vertical obstacle to the flow around the wing tip. Sometimes additional fuel tanks are mounted there.

Each wing tip generates its own vortex. Energetically, it is more economical to have many small vortices instead of one large one because the losses are proportional to the square of the vortex size. It is easy to see that the sum of the squares is smaller than the square of the sum. Biplanes (having four wing tips) have less induced drag than monoplanes, everything else being the same. Soaring birds reduce the induced drag by spreading the feathers so that, near the end of the wing, there are many tips.

Obviously the smaller the chord at the tip of the wing, the smaller the induced drag. In a rectangular wing of a given area, a larger length, H, results in a smaller chord, K. The wing has a larger aspect ratio and, consequently a smaller induced drag.

Tapered wings have smaller wing-tip chords than rectangular wings of the same area and, again, have smaller induced drag.

Gliders have long, slender wings to maximize the aspect ratio (minimizing the induced drag). In fast moving airplanes, the **parasitic drag**† is dominant, making the induced drag unimportant: fast planes can tolerate small aspect ratios.

When a wing is tested, the total drag measured includes the induced drag, hence it is customary to indicate the aspect ratio, $\mathcal{R}$, of the test section when aerodynamic coefficients are tabulated.

† Parasitic drag is caused by parts of the machine that offer resistance to the flow of air but do not generate lift.

15.10 Wind Turbine Analysis

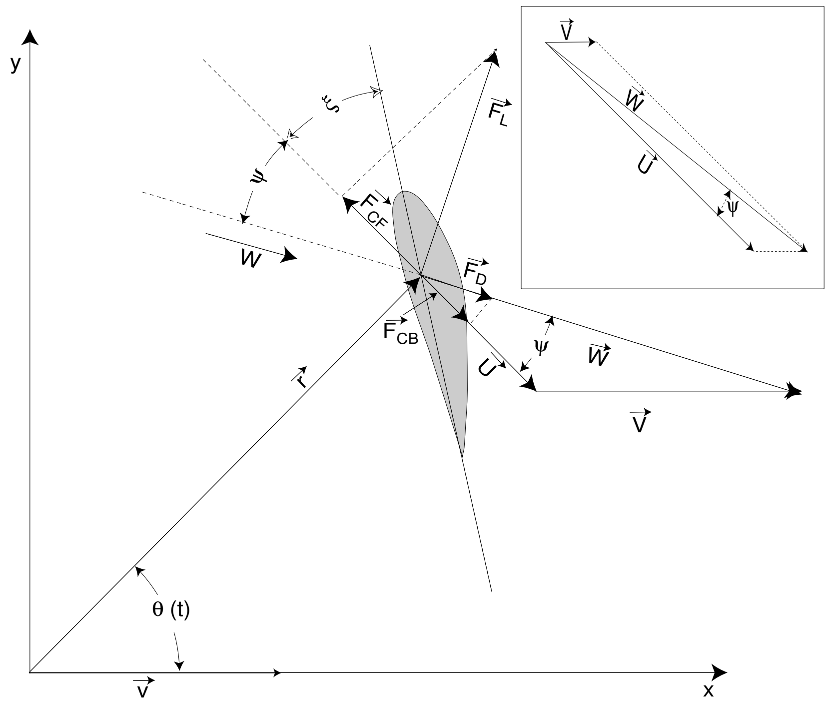


Figure 15.15 Angles and forces on a wing. Usually, $\vec{U}$, is much larger than $\vec{V}$, but for clarity in the drawing (but not in the inset), they were taken as approximately equal. This exaggerates the magnitude of the angle, ψ. The inset shows the wind vectors.

> *As an example of wind turbine analysis, we have chosen a gyromill because, even though this type of turbine is not in use, its analysis is much simpler than that of, say, propeller-type machines. It, nevertheless brings out important conclusions that, with suitable modifications, are also applicable to other lift-type devices.*
>
> *For those interested in an introductory analysis of propeller turbines, we recommend reading the book by Duncan. See References.*

Consider a vertical-axis wind turbine of either the Darrieus or the Mcdonnell-Douglas type. Consider also a right-handed orthogonal coordinate system with the z-axis coinciding with the vertical axis of the machine. The system is so oriented that the horizontal component of the wind velocity, V, is parallel to the x-axis.

We will assume that the airfoil, whose cross section lies in the x-y plane, has a chord that makes an angle, ξ, with the normal to the radius vector. This is the **setup** angle chosen by the manufacturer of the wind turbine. It may be adjustable and it may even vary during one rotation. In this analysis, it is taken as a constant.

The radius vector makes an angle, $\theta(t)$, with the x-axis. Figure 15.15 indicates the different angles and vectors involved in this derivation. The inset shows how the wind velocity, V, and the velocity, U (owing to the rotation) combine to produce an **induced** velocity, W which is the actual air velocity perceived by the wing. U is, of course, the velocity the wing would perceive if there were no wind.

Refer to Figure 15.15. The angle of attack is $\alpha = \psi + \xi$.

$$\vec{\omega} = \vec{k}\ \omega, \tag{27}$$

$$\vec{r} = r(\vec{i}\ \cos\theta + \vec{j}\ \sin\theta), \tag{28}$$

$$\vec{U} = -\vec{\omega}\ \times\ \vec{r} = -\begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & \omega \\ r\ \cos\theta & r\ \sin\theta & 0 \end{pmatrix} = U\ (\vec{i}\ \sin\theta - \vec{j}\ \cos\theta), \tag{29}$$

$$\vec{W} = \vec{U} + \vec{V} = \vec{i}(V + U\ \sin\theta) - \vec{j}\ U\ \cos\theta, \tag{30}$$

$$\begin{aligned} W &= \sqrt{V^2 + U^2\sin^2\theta + 2UV\sin\theta + U^2\cos^2\theta} \\ &= \sqrt{V^2 + U^2 + 2UV\sin\theta} \equiv \Gamma V, \end{aligned} \tag{31}$$

where

$$\Gamma \equiv \sqrt{1 + \frac{U^2}{V^2} + 2\frac{U}{V}\sin\theta}, \tag{32}$$

$$\begin{aligned} \vec{U}\cdot\vec{W} &= (U\ \sin\theta)(V + U\ \sin\theta) + U^2\cos^2\theta \\ &= U^2 + UV\ \sin\theta = UW\ \cos\psi, \end{aligned} \tag{33}$$

from which

$$\cos\psi = \frac{U^2 + UV\ \sin\theta}{UW} = \frac{U + V\ \sin\theta}{\sqrt{V^2 + U^2 + 2UV\ \sin\theta}} = \frac{U/V + \sin\theta}{\Gamma}. \tag{34}$$

For a given wind speed, V, and angular velocity, ω, of the wind turbine, the ratio, U/V, is constant. The quantity, Γ, and the angle, ψ, vary with the angular position of the wing, thus they vary throughout the revolution. Consequently, the angle of attack also varies. Clearly, if there is no wind, α is constant. If there is a high U/V ratio (if the wind speed is much smaller than that of the rotating wing), then α varies only a little throughout the revolution.

Given a U/V ratio and a wind velocity, it is possible to calculate both α and W for any wing position, θ. For a given W and α, the wing will generate a lift

$$F_L = \tfrac{1}{2}\rho W^2 A_p C_L, \tag{35}$$

and a drag

$$F_D = \tfrac{1}{2}\rho W^2 A_p C_D.$$

In the preceding equations, A_p is the area of the wing and F stands for the force on the wing.

Note that $\vec{F}_L$ is normal to $\vec{W}$ in the x-y plane and that $\vec{F}_D$ is parallel to $\vec{W}$. The lift force, $\vec{F}_L$, has a component, $\vec{F}_{CF}$, normal to the radius vector, causing a forward torque. The drag force, $\vec{F}_D$, has a component, $\vec{F}_{CB}$, also normal to the radius vector, causing a retarding torque.

The resulting torque is

$$\Upsilon = r(F_{CF} - F_{CB}). \tag{36}$$

From Figure 15.15,

$$F_{CF} - F_{CB} = F_L \sin\psi - F_D \cos\psi = \tfrac{1}{2}\rho W^2 A_p (C_L \sin\psi - C_D \cos\psi). \tag{37}$$

Thus,

$$\begin{aligned} \Upsilon &= \tfrac{1}{2}\rho W^2 A_p r (C_L \sin\psi - C_D \cos\psi) \\ &= \tfrac{1}{2}\rho V^2 A_p r \left[\Gamma^2 (C_L \sin\psi - C_D \cos\psi)\right]. \end{aligned} \tag{38}$$

The average torque taken over a complete revolution is

$$<\Upsilon>= \frac{1}{2\pi}\int_0^{2\pi} \Upsilon(\theta)\ d\theta. \tag{39}$$

In the expression for Υ, only the part in brackets is a function of θ. Let us define a quantity, D:

$$D \equiv \Gamma^2 (C_L \sin\psi - C_D \cos\psi), \tag{40}$$

$$<D>= \frac{1}{2\pi}\int_0^{2\pi} D\ d\theta, \tag{41}$$

$$<\Upsilon>= \tfrac{1}{2}\rho V^2 A_p r <D>. \tag{42}$$

The power delivered by the turbine to its load is

$$P_D = \omega <\Upsilon> N. \tag{43}$$

Here, N is the number of wings on the wind turbine. The swept area is (see Figure 15.16)

$$A_v = 2rH, \tag{44}$$

and the area of each wing is

$$A_p = KH, \tag{45}$$

where H is the (vertical) length of the wing and K is the chord (assumed uniform).

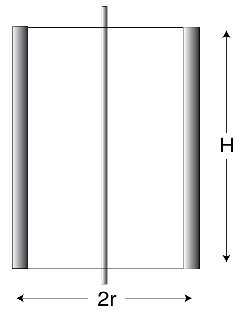


Figure 15.16 The aspect ratio of a wind turbine.

A **solidity**, S, is defined:

$$S = \frac{NA_p}{A_v} = N\frac{K}{2r} \tag{46}$$

The **available** power from the wind is

$$P_A = \frac{16}{27}\tfrac{1}{2}\rho V^3 A_v, \tag{47}$$

$$\eta = \frac{P_D}{P_A} = \frac{\frac{1}{2}\rho V^2 N A_p r\omega <D>}{\frac{1}{2}\rho V^3 A_v \frac{16}{27}} = \frac{27}{16}\frac{U}{V} <D> S. \tag{48}$$

The efficiency formula derived above is correct only to first order. It ignores parasitic losses owing to friction and to the generation of vortices; it disregards the reduction in wind velocity caused by the wind turbine itself; it fails to take into account the interference of one wing blade on the next. In fact, Equation 48 predicts that with large enough solidities, the efficiency can exceed unity. We will discuss this question a little later.

Notice that the $\frac{U}{V} <D>$ product is a function of the parameter U/V.

$<D>$ must be obtained from numerical analysis looking up values of C_L and C_D for the various α that appear during one revolution.

To gain an idea of the shape of the $\frac{U}{V} <D>$ vs U/V graph, consider the situation when $U = 0$. Clearly, $U/V = 0$ and since D cannot be infinite, $\frac{U}{V} <D>$ must also be 0.[†]

[†] When $U = 0$, Γ is unity (Equation 32) and, from Equation 34, $\cos\psi = \sin\theta$ and $\sin\psi = \cos\theta$. This makes

$$D = C_L \cos\theta - C_D \sin\theta, \tag{49}$$

and consequently, provided C_L and C_D are constant

$$<D> = 0, \tag{50}$$

because the mean value of $\sin\theta$ and of $\cos\theta$ is zero. Since the torque is proportional to $< D >$, this type of wind turbine has no torque when stalled: it has zero starting torque and requires a special starting arrangement (such as a small Savonius on the same shaft). Actually, C_L and C_D do depend on Θ, and Equation 49 holds only approximately.

When $U \to \infty$, $W \to U$ and $\psi \to 0$. From Equation 40,

$$D = \Gamma^2(C_L \sin\psi - C_D \cos\psi) \to -\Gamma^2 C_D,$$

thus, for large values of U/V, $D < 0$ and, consequently, $\frac{U}{V} <D>< 0$.

This means that, at high rpm, the wind turbine has a negative torque and tends to slow down. One can, therefore, expect that the efficiency has a maximum at some value of U/V in the range $0 < U/V < \infty$.

As an example, we have computed $\frac{U}{V} <D>$ for various values of U/V and a number of setup angles, ξ. The airfoil used was the Gö-420. The results are shown in Figure 15.17. It can be seen that the optimum setup angle (the one that leads to the highest $\frac{U}{V} <D>$) is $-6°$. Symmetric airfoils work best with $\xi = 0$.

For $\xi = -6°$, the airfoil reaches a $\frac{U}{V} <D>$ of 4.38 (non-dimensional) at a U/V of 6.5. Thus, in this particular case, the efficiency formula yields:

$$\eta_{max} = 7.39S. \tag{51}$$

Were one to believe the formula above, efficiencies greater than 1 could be reached by using solidities, S, larger than 0.135. Clearly, there must be some value of solidity above which the formula breaks down. In Figure 15.18, the efficiency of a wind turbine is plotted versus solidity. The linear dependence predicted by Equation 51 is represented by the dashed line with the 7.39 slope of our example. Using a more complicated aerodynamical model, Sandia obtained the results shown in the solid line. It can be seen that increasing the solidity beyond about 0.1 does not greatly affect the efficiency. One can distinguish two regions in the efficiency versus solidity curve: one in which, as predicted by our simple derivation, the efficiency is proportional to the solidity, and one in which the efficiency is (roughly) independent of the solidity.

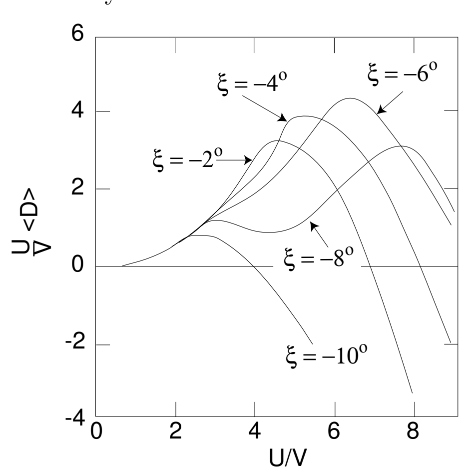


Figure 15.17 Performance of the Gö-420 airfoil in a vertical-axis turbine.

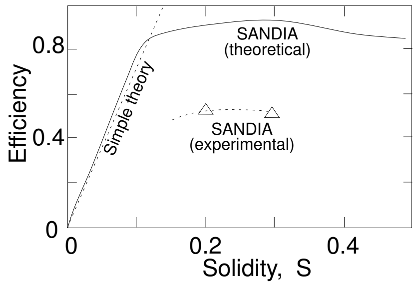


Figure 15.18 Effect of solidity on efficiency of a vertical-axis wind turbine.

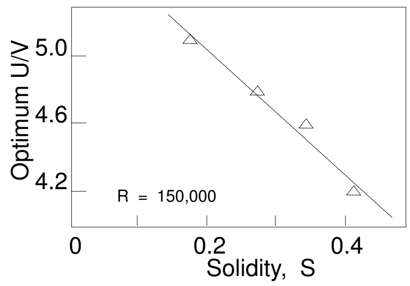


Figure 15.19 Dependence of the optimum U/V on solidity. Experimental data from the Sandia 2-m diameter Darrieus turbine.

Triangles in Figure 15.18 indicate values of efficiency measured by Sandia using small models of the wind turbines. Measured efficiencies are about half of the calculated ones. This discrepancy is discussed further on.

The main reason for the behavior depicted in Figure 15.18 is that our simple theory failed to account for the interference of one wing with the next. The larger the solidity, the farther the disturbance trails behind the wing and the more serious the interference thus counteracting the efficiency gain from a larger S. Consequently, the optimum $\frac{U}{V}$ shrinks as S increases.

Figure 15.19 shows the experimentally determined effect of solidity on the optimum $\frac{U}{V}$. If the straight line were extrapolated, one would conclude that for $S = 1$, the optimum $\frac{U}{V}$ would be about 0.7.

In the range of solidities that have only a small effect on the efficiency, increasing S results in a wind turbine that rotates more slowly (because of the smaller optimum U/V) and has more torque (because the efficiency—and consequently the power—is the same). Increasing S has the effect of "gearing down" the wind turbine. Since the cost of a wind turbine is roughly proportional to its mass, and hence to its solidity, one should prefer machines with S in the lower end of the range in which it does not affect the efficiency. That is why, for large machines, propellers are preferred to vanes. Nevertheless, for small wind turbines, the simplicity of vane construction may compensate for the larger amount of material required.

Although the equations derived above will help in the understanding of the basic wind turbine processes, they fall far short from yielding accurate performance predictions. Numerous refinements are needed:

1. Frictional losses in bearings must be taken into account.
2. The rotating wings create vortices that represent useless transformation of wind energy into whirling motion of the air. The effect of such vortices has to be considered.
3. The wind, having delivered part of its energy to the machine, must nec-

essarily slow down. Thus, the average wind velocity seen by the blades is less than the free stream velocity and the power is, correspondingly less than that predicted by the formulas.

Single streamtube models are based on an average wind slowdown. These models ignore the nonuniformity of the wind velocity in the cross section of the wind turbine. By contrast, if the wind slowdown is considered in detail, then we have a **multiple streamtube** model.

Figure 15.20 shows how the ratio of the streamtube to freestream velocity varies with position in the Sandia 2-m diameter Darrieus turbine. When the calculations are based on the multi streamtube model, the predicted performance approaches reality as can be seen in Figure 15.21 in which the measured efficiency (squares) is compared with the values predicted using both the single and the multi-streamtube models. Even the multi-streamtube model overestimates the performance.

4. The accuracy of the prediction is improved by using the appropriate Reynolds number, a quantity that actually varies throughout the revolution. Figure 15.22 shows the effect of the Reynolds number on the performance of the wind turbine. As expected, the measurements, made at Sandia, show that the larger R, the better the performance.

Clearly, a refined wind turbine performance model is too complicated to be treated in this book.

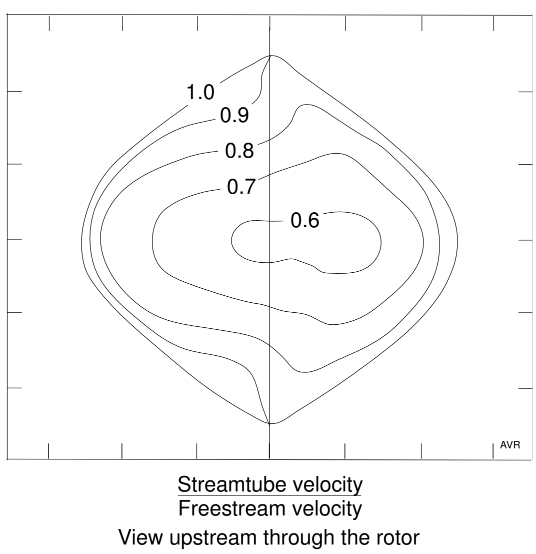


Figure 15.20 Streamtube velocity through the rotor of a Sandia 2-m Darrieus turbine.

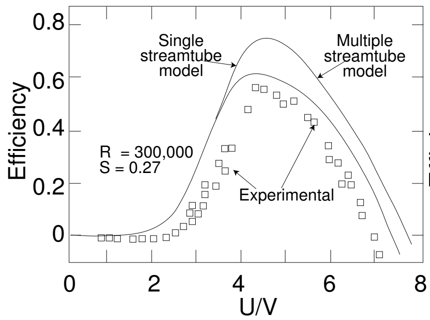


Figure 15.21 Calculated efficiencies compared with observed values. Data from Sandia.

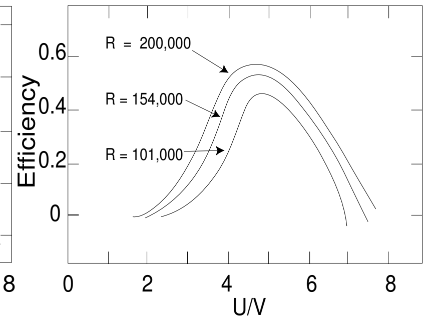


Figure 15.22 Influence of the Reynolds number on the performance. Data from Sandia.

15.11 Aspect Ratio (of a wind turbine)

In Figure 15.16 (previous section), the aspect ratio, $\mathit{A\!\!R}_{turb}$, of a Gyromill (McDonnell-Douglas vertical-axis wind turbine) was defined:

$$\mathit{A\!\!R}_{turb} = \frac{H}{2r} = \frac{A_v}{4r^2} = \frac{H^2}{A_v}. \tag{52}$$

This is, of course, different from the aspect ratio of the wing, which is

$$\mathit{A\!\!R}_w = \frac{H}{K}, \tag{53}$$

where K is the chord length of the wing. Since the solidity is $S = NK/2r$, the wing aspect ratio can be rewritten as $\mathit{A\!\!R}_w = HN/2rS$. However, the aspect ratio of the wind turbine is $\mathit{A\!\!R}_{turb} = H/2r$, hence,

$$\mathit{A\!\!R}_{turb} = \frac{S}{N}\mathit{A\!\!R}_w. \tag{54}$$

For constant N and constant S (constant first-order efficiency), the aspect ratio of the wind turbine is proportional to that of the wing. Wingtip drag decreases with increasing wing aspect ratio, consequently, wind turbine efficiencies actually tend to go up with increasing $\mathit{A\!\!R}_{turb}$.

From Equation 46, it can be seen that if both the swept area, A_v, (power) and the solidity, S, (efficiency) are kept constant, then, to a first approximation, the mass of the wings remains constant because their area, A_p, is constant. However, larger aspect ratios require higher towers. The mass of the tower increases with height faster than linearly because higher towers must have a larger cross-section. On the other hand, the struts that support the wings become smaller with increasing $\mathit{A\!\!R}_{turb}$, again nonlinearly.

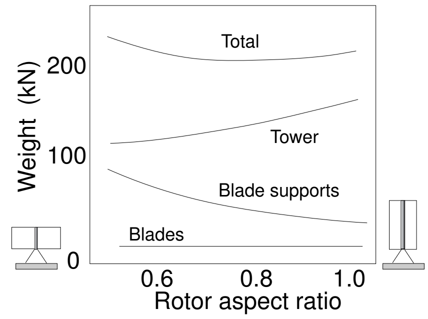


Figure 15.23 Influence of the aspect ratio on the mass of a wind turbine. McDonnell-Douglas120 kW Gyromill.

Thus, we have a situation like the one depicted in Figure 15.23 which displays data for the McDonnell-Douglas "gyromill." There is a minimum in total mass of the wind turbine at a $\mathcal{AR}_{turb}$ of, roughly, 0.8.

15.12 Centrifugal Force

Consider a section of a wing with mass M. Its weight is Mg (g is the acceleration of gravity). The centrifugal force acting on the section is

$$F_c = \frac{MU^2}{r} = M\omega^2 r. \tag{55}$$

The ratio of centrifugal force to weight is

$$\frac{F_c}{W} = \frac{\omega^2 r}{g}. \tag{56}$$

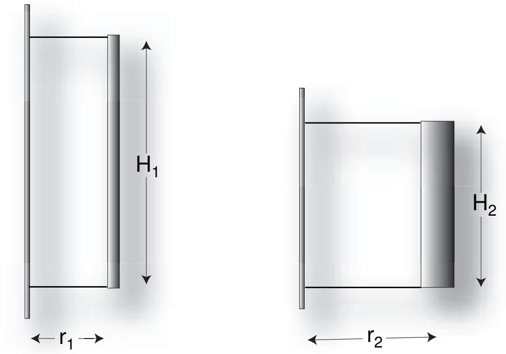


Figure 15.24 Two wind turbines with the same area, but with different aspect ratios.

In a 10-m diameter wind turbine rotating at 100 rpm ($\omega = 10.5$ rad/s), a section of the wing will experience a centrifugal force 55 times larger than its weight. This illustrates the need for careful design to minimize the centrifugal effects. For instance, what aspect ratio minimizes these effects?

Compare two wind turbines with the same swept area. If they have the same solidity and use the same airfoil, they will deliver the same power when operated at the same U/V ratio. If they have different aspect ratios then they must have:

$$r_1 H_1 = r_2 H_2. \tag{57}$$

The centrifugal force per unit length is

$$F_c = \frac{MU^2}{r}, \tag{58}$$

where M is the mass of the wing per unit length. The ratio of the centrifugal forces on the two wind turbines is

$$\frac{F_{c_1}}{F_{c_2}} = \frac{M_1 r_2}{M_2 r_1}. \tag{59}$$

If the two wings have a solid cross section, then the masses (per unit length) are proportional to the square of the chord, K. In general,

$$M = bK^a, \tag{60}$$

where a is an exponent that depends on the type of construction. From this

$$K = \left(\frac{M}{b}\right)^{1/a}. \tag{61}$$

Since the wind turbines have the same solidity, $NK/2r$,

$$\frac{N_1}{2r_1}\left(\frac{M_1}{b}\right)^{1/a} = \frac{N_2}{2r_2}\left(\frac{M_2}{b}\right)^{1/a}, \tag{62}$$

$$\frac{M_1}{M_2} = \left(\frac{N_2 r_1}{N_1 r_2}\right)^{a}, \tag{63}$$

$$\frac{F_{c_1}}{F_{c_2}} = \left(\frac{N_2}{N_1}\right)^{a}\left(\frac{r_1}{r_2}\right)^{a-1}, \tag{64}$$

and, if the two wind turbines have the same number of wings,

$$\frac{F_{c_1}}{F_{c_2}} = \left(\frac{r_1}{r_2}\right)^{a-1}. \tag{65}$$

15.32

From the preceding formula, it becomes clear that for any $a > 1$, the larger the radius, the larger the centrifugal force. Since it is difficult to achieve low a's, this favors large aspect ratios.

The centrifugal force in a "gyromill" poses a difficult problem for the wind turbine designer. However, as explained in Subsection 15.2.2, this problem disappears in the case of the Darrieus (egg-beater) configuration.

15.13 Performance Calculation

Consider a vertical-axis wind turbine with the characteristics given in the table below.

Table 15.1
Wind Turbine Characteristics

Number of wings	$N = 3$
Height (length of wings)	$H = 16$ m
Radius	$r = 10$ m
Solidity	$S = 0.27$
Site	sea level
Wing performance	see Figure 15.25 and Table 15.2.

Assume that the performance data are valid for the actual Reynolds number. The wind turbine drives a load whose torque obeys the relationship

$$\Upsilon_L = 2000\omega. \tag{66}$$

The performance of the wind turbine is presented in the form of a torque vs angular velocity plot. A different plot must be constructed for each wind velocity of interest.

Take, for instance, $V = 10$ m/s. The questions to be answered are:

1. What is the operating rpm?
2. What is the power delivered to the load?
3. What is the actual (average) Reynolds number?
4. What is the efficiency of the wind turbine?

Using the technique described in Section 15.10, a plot of efficiency versus U/V is constructed. Such a plot (similar to those in Figure 15.17) is shown in Figure 15.25. The corresponding numerical data appear in Table 15.2.

Now, let us construct the torque versus angular velocity plot:

$$\omega = \frac{U}{r} = \frac{U}{V} \times \frac{V}{r} = \frac{U}{V}. \tag{67}$$

Observe that in this particular example, the value of V/r is 1.

$$A_v = 2rH = 320 \text{ m}^2, \tag{68}$$

$$\rho = 1.29 \text{ kgm}^{-3} \text{ (sea level)}, \tag{69}$$

$$P_D = \frac{16}{27}\,\tfrac{1}{2}\rho V^3 A_v \eta = 122,000\ \eta, \tag{70}$$

$$\Upsilon = \frac{P_D}{\omega} = 122,000 \frac{\eta}{U/V}. \tag{71}$$

Notice that, Υ, above, is the total torque of the wind turbine, not the torque per wing, as before.

Using the performance table (Table 15.2), it is easy to calculate η for any U/V (or for any ω). The plot of Υ versus ω is shown in Figure 15.26. In the same figure, the Υ_L versus ω characteristic of the load is plotted. It can be seen that two different values of ω yield wind turbine torques that match that of the load. In point A, if there is a slight increase in wind velocity, the wind turbine will speed up, its torque will become larger and the turbine will accelerate, increasing the torque even further. It is an unstable point. In point B, an increase in wind velocity will reduce the wind turbine torque causing it to slow down back to its stable operating point.

At the stable point, B, the angular velocity in our example is 5.4 rad/s, equivalent to 0.86 rps or 52 rpm. The power generated will be

$$P = \Upsilon\omega = 2000\ \omega^2 = 58,300 \text{ W}. \tag{72}$$

Table 15.2
Efficiency Versus U/V for the Wing in the Example

U/V	Efficiency
2.0	0.00
2.2	0.02
2.4	0.05
2.6	0.08
2.8	0.12
3.0	0.15
3.5	0.33
4.0	0.49
4.5	0.56
5.0	0.53
5.5	0.44
6.0	0.33
6.5	0.17
7.0	0.00

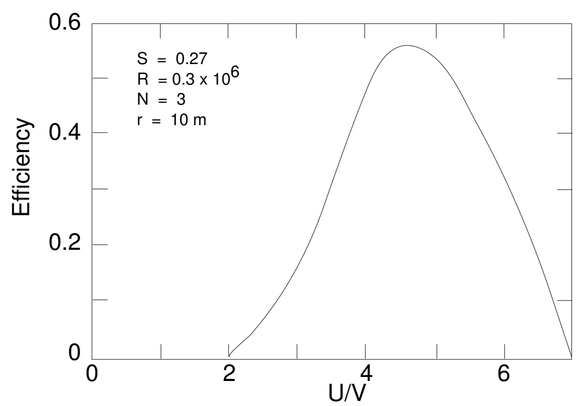


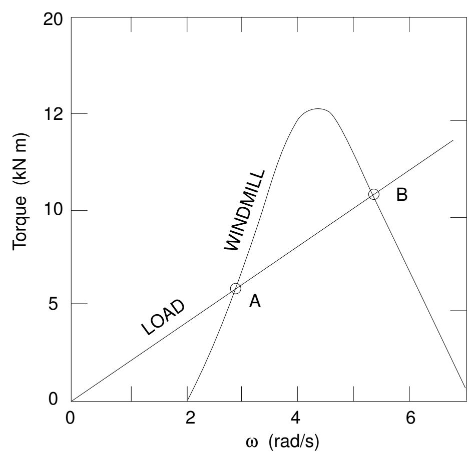
Figure 15.25 Efficiency versus U/V for the wing in the example.

The efficiency that corresponds to $U/V = \omega = 5.4$ is 0.46.
The solidity is

$$S = \frac{NK}{2r} \qquad \therefore \qquad K = \frac{2rS}{N} = 1.8 \text{ m}. \tag{73}$$

The Reynolds number is

$$R = 70,000\ WK \approx 70,,,,,000\ UK = 6.8 \times 10^6. \tag{74}$$



15.14 Magnus Effect

In the introduction to this chapter, we made reference to the use of rotating cylinders to generate lift useful in driving wind turbines. The lift is caused by the **Magnus** effect, used by baseball pitchers when throwing a "curve."

Consider a rotating cylinder exposed to a stream of air.

Sufficiently far away from the cylinder, the air is undisturbed—that is, it moves with the velocity of the wind.

Immediately in contact with the cylinder, at point a (see Figure 15.26), the air flows from right to left in a direction opposite to that of the wind because the (rough) cylinder surface exerts friction on the air and accelerates it in the direction of its own motion. Its velocity is equal to the linear velocity of the cylinder surface. A gradient of velocity is established as illustrated in the figure.

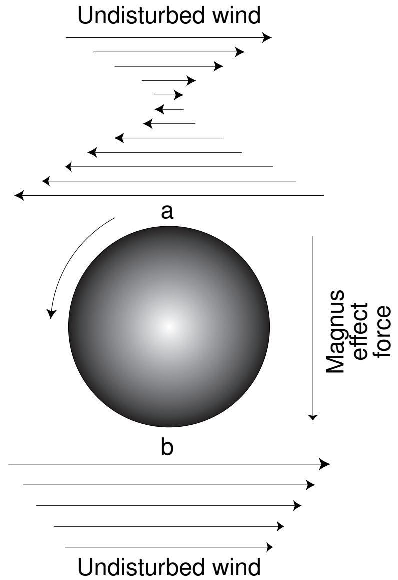


Figure 15.26 A rotating cylinder induces a velocity gradient in the wind from which a lateral force results.

On the opposite side, b, the cylinder impels the air from left to right causing it to flow faster than the stream. Again a velocity gradient may be established.

The average stream velocity is larger on the b-side than on the a-side of the cylinder.

According to Bernoulli's law, the high velocity side , b, experiences a smaller pressure than the low velocity side, a. Consequently, a force from a to b is exerted on the cylinder. This force, the result of the aerodynamic reaction on a rotating body, is called the **Magnus effect**. It is proportional to $\vec{\Omega}\times\vec{V}$, where $\vec{\Omega}$ is the angular velocity of the cylinder and $\vec{V}$ is the velocity of the wind.

References

Duncan, W. J., A. S. Thom, and A. D, Young, *An Elementary Treatise on the Mechanics of Fluids* The English Language Book Society, **1960**.

Gipe, Paul, Wind energy comes of age, *John Wiley*, **1995**.

PROBLEMS

Prob 15.1 At a given sea-level location, the wind statistics, taken over a period of one year and measured at an anemometer height of 10 m above ground, are as follows:

Number of hours	**Velocity** (m/s)
90	25
600	20
1600	15
2200	10
2700	5
remaining time	calm

The velocity is to be assumed constant in each indicated range (to simplify the problem).

Although the wind varies with the 1/7th power of height, assume that the velocity the windmill sees is that at its center.

The windmill characteristics are:

Efficiency	70%
Cost	150 $\$/\text{m}^2$
Weight	100 kg/m^2

The areas mentioned above are the swept area of the windmill.

If h is the height of the tower and M is the mass of the windmill on top of the tower, then the cost, C_T, of the tower is

$$C_T = 0.05\ h\ M.$$

How big must the swept area of the windmill be so that the average delivered power is 10 kW? How big is the peak power delivered—that is, the rated power of the generator? Be sure to place the windmill at the most economic height. How tall will the tower be?

Neglecting operating costs and assuming an 18% yearly cost of the capital invested, what is the cost of the MWh?

If the windmill were installed in La Paz, Bolivia, a city located at an altitude of 4000 m, what would the average power be, assuming that the winds had the velocity given in the table above? The scale height of the atmosphere is 8000 meters (i.e., the air pressure falls exponentially with height with a characteristic length of 8000 m).

Prob 15.2 A utilities company has a hydroelectric power plant equipped with generators totaling 1 GW capacity. The utilization factor used to be exactly 50%—that is, the plant used to deliver every year exactly half of the energy the generators could produce. In other words, the river that feeds the plant reservoir was able to sustain exactly the above amount of energy. During the wet season, the reservoir filled but never overflowed.

Assume that the plant head is an average of 80 m and that the plant (turbines and generators) has an efficiency of 97%.

What is the mean rate of flow of the river (in m^3/s)?

With the industrial development of the region, the utilities company wants to increase the plant utilization factor to 51%, but, of course, there is not enough water for this. So, they decided to use windmills to pump water up from the level of the hydraulic turbine outlet to the reservoir (up 80 m).

A careful survey reveals that the wind regime is the one given in the table below:

¡v¿ (m/s)	Θ
5	0.15
7	0.45
10	0.30
12	0.10

$<v>$ is the mean cubic wind velocity and θ is the percentage of time during which a given value of $<v>$ is observed. The generator can be dimensioned to deliver full power when $<v>=$ 12 m/s or when $<v>$ is smaller. If the generator is chosen so that it delivers its rated full power for, say, $<v>=$ 10 m/s, then a control mechanism will restrict the windmill to deliver this power even if $<v>$ exceeds the 10 m/s value.

Knowing that the cost of the windmill is \$10 per m^2 of swept area, that of the generator is \$0.05 per W of rated output, the efficiency of the windmill is 0.7 and that of the generator is 0.95, calculate which is the most economic limiting wind velocity.

What is the swept area of the windmill that will allow increasing the plant factor to 51% ? (The pumps are 95% efficient.) What is the cost of the MWh generated by the windmills, assuming an annual cost of investment of 20% and neither maintenance nor operating costs.

The windmills are, essentially, at sea level.

Prob 15.3 A windmill is installed at a sea-level site where the wind has the following statistics:

v (m/s)	% of time
0	30
3	30
9	30
12	8
15	2

The velocity in the table above should be the mean cubic velocity of the wind, however, to simplify the problem, assume that the wind actually blows at a constant 3 m/s 30% of the time, a constant 9 m/s another 30% of the time and so on.

The windmill characteristics are:

Efficiency (including generator)	0.8
Windmill cost	200 $/m^2$ of swept area
Generator cost	200 $/kW of rated power.

Rated power is the maximum continuous power that the generator is supposed to deliver without overheating. The duty cycle is 1—that is, the windmill operates continuously (when there is wind) throughout the year. Consider only investment costs. These amount to 20% of the investment, per year.

The system can be designed so that the generator will deliver full (rated) power when the wind speed is 15 m/s. The design can be changed so that rated power is delivered when the wind speed is 12 m/s. In this latter case, if the wind exceeds 12 m/s, the windmill is shut down. It also can be designed for rated power at 9 m/s, and so on.

We want a windmill that delivers a maximum of 1 MW. It has to be designed so that the cost of the generated electricity over a whole year is minimized. What is the required swept area? What is the cost of electricity?

Prob 15.4 *For this problem, you need a programmable calculator or a computer*

Consider an airfoil for which

$C_L = 0.15\alpha,$

$C_D = 0.015 + 0.015|\alpha|,$

for $-15° < \alpha < 15°$ where α is the angle of attack. A wing with this airfoil is used in a vertical-axis windmill having a radius of 10 m. The setup angle is 0.

Tabulate and plot α as well as the quantity D (see text) as a function of θ, for $U/V = 6$. Use increments of θ of 30° (i.e., calculate 12 values).

Considerations of symmetry facilitate the work. Be careful with the correct signs of angles. It is easy to be trapped in a sign error. Find the mean value of $< D >$.

Prob 15.5 In the region of Aeolia, on the island of Anemos, the wind has a most peculiar behavior. At precisely 0600, there is a short interval with absolutely no wind. Local peasants set their digital watches by this lull. Wind velocity then builds up linearly with time, reaching exactly 8 m/s at 2200. It then decays, again linearly, to the morning lull.

A vertical-axis windmill with 30 m high wings was installed in that region. The aspect ratio of the machine is 0.8 and its overall efficiency is 0.5. This includes the efficiency of the generator.

What is the average power generated? What is the peak power? Assuming a storage system with 100% turnaround efficiency, how much energy must be stored so that the system can deliver the average power continuously? During what hours of the day does the windmill charge the storage system?

Notice that the load always gets energy from the storage system. This is to simplify the solution of the problem. In practice, it would be better if the windmill fed the load directly and only the excess energy were stored.

Prob 15.6 A vertical-axis windmill with a rectangular swept area has an efficiency whose dependence on the U/V ratio (over the range of interest) is given, approximately, by

$$\eta = 0.5 - \frac{1}{18}\left(\frac{U}{V} - 5\right)^2.$$

The swept area is 10 m^2 and the aspect ratio is 0.8.

The wind velocity is 40 km/h.

What is the maximum torque the windmill can deliver? What is the number of rotations per minute at this torque? What is the power delivered at this torque? What is the radius of the windmill and what is the height of the wings? If the windmill drives a load whose torque is given by

$$\Upsilon_L = \frac{1200}{\omega} \qquad \text{Nm}$$

where ω is the angular velocity, what is the power delivered to the load? What are the rpm when this power is being delivered?

Prob 15.7 An engineering firm has been asked to make a preliminary study of the possibilities of economically generating electricity from the winds in northeastern Brazil. As a first step, a quick and very rough estimate is required. Assume that the efficiency can be expressed by the ultra-simplified formula of Equation 48 in the text. The results will be grossly over-optimistic because we fail to take into account a large number of loss mechanisms and we assume that the turbine will always operate at the best value of $\frac{U}{V}D$, which, for the airfoil in question is 4.38. Our first cut will lead to unrealistic results but will yield a ball park idea of the quantities involved.

In the region under consideration, the trade winds blow with amazing constancy, at 14 knots. Assume that this means 14 knots at a 3 m anemometer height. The wind turbines to be employed are to have a rated power of 1 MWe (MWe = MW of electricity).

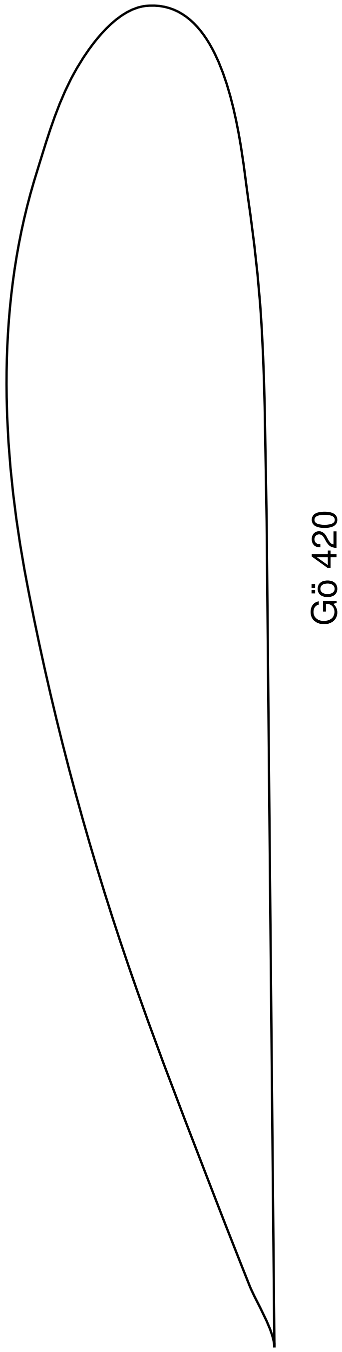
For the first cut at the problem we will make the following assumptions:

a. The configuration will be that of the McDonnell-Douglas gyromill, using three wings.
b. Wind turbine efficiency is 80%.
c. The wings use the Göttingen-420 airfoil (see drawing at the end of the problem). We will accept the simple efficiency formula derived in this chapter.
d. The wind turbine aspect ratio will be 0.8.
e. The wings will be hollow aluminum blades. Their mass will be 25% of the mass of a solid aluminum wing with the same external shape. Incidentally, aluminum is not a good choice for wind turbine wings because it is subject to fatigue. Composites are better.
f. The total wind turbine mass will be 3 times the mass of the three wings taken together.
g. Estimated wind turbine cost is $1.00/kg. Notice that the cost of aluminum was $1200/kg in 1852, but the price is now down to $0.40/kg.
h. The cost of investment is 12% per year.

Estimate:

1. The swept area.
2. The wing chord.
3. The wing mass.
4. The Reynolds number when the turbine is operating at its rated power. Assume that this occurs at the optimum U/V ratio.
5. The rpm at the optimum U/V ratio.
6. The torque under the above conditions.
7. The tension on each of the horizontal supporting beams (two beams per wing).
8. The investment cost per rated kW. Assuming no maintenance nor operating cost, what is the cost of the generated kWh? Use a utilization factor of 25%.

15.41



15.42

Prob 15.8 A sailboat has a drag, F, given by

$$F = aW^2,$$

where F is in newtons, W is the velocity of the boat with respect to the water (in m/s), and $a = 80$ kg/m.

The sail of the boat has a 10 m^2 area and a drag coefficient of 1.2 when sailing before the wind (i.e., with a tail-wind).

Wind speed is 40 km/h.

When sailing before the wind, what is the velocity of the boat? How much power does the wind transfer to the boat? What fraction of the available wind power is abstracted by the sail?

Prob 15.9 A vertical-axis windmill of the gyromill configuration extracts (as useful power) 50% of the available wind power. The windmill has a rectangular swept area with a height, H, of 100 m and an aspect ratio of 0.8.

The lower tips of the wings are 10 m above ground. At this height, the wind velocity is 15 m/s. It is known that the wind increases in velocity with height according to the 1/7th power law.

Assuming that the windmill is at sea level, what power does it generate?

Prob 15.10 *Solve equations by trial and error. Use a computer*

Can a wind-driven boat sail directly into the wind? Let's find out (forgetting second order effects).

As a boat moves through the water with a velocity, W (relative to the water), a drag force, F_w, is developed. Let $F_w = 10W^2$.

The boat is equipped with a windmill having a swept area of 100 m^2 and an overall efficiency of 50%. The power generated by the windmill is used to drive a propeller which operates with 80% efficiency and creates a propulsive force, F_p, that drives the boat.

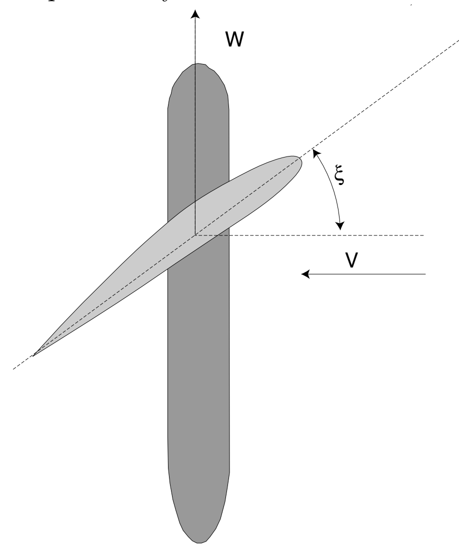
The windmill is oriented so that it always faces the *induced* wind, i.e., the combination of V and W.

The wind exerts a force, F_{WM}, on the windmill. This force can be estimated by assuming a $C_D = 1.1$ and taking the swept area as the effective area facing the wind.

Wind velocity, V, is 10 m/s. What is the velocity, W_S, of the boat in the water? Plot W as a function of the angle, ϕ, between the direction of the wind and that of the boat. In a tail wind, $\phi = 0$ and in a head wind, $\phi = 180°$.

The boat has a large keel, so that the sideways drift caused by the "sail" effect of the windmill is negligible.

Prob 15.11 *Solve equations by trial and error. Use a computer*



A vehicle is mounted on a rail so that it can move in a single direction only. The motion is opposed by a drag force, F_W,

$$F_W = 100W + 10W^2$$

where W is the velocity of the vehicle along the rail. Notice that the drag force above does not include any aerodynamic effect of the "sail" that propels the vehicle. Any drag on the sail has to be considered in addition to the vehicle drag.

The wind that propels the vehicle is perpendicular to $\vec{W}$, and has a velocity, $V = 10$ m/s. It comes from the starboard side (the right side of the vehicle).

The "sail" is actually an airfoil with an area of 10.34 m^2. It is mounted vertically, i.e., its chord is horizontal and its length is vertical. The reference line of the airfoil makes an angle, ξ, with the normal to $\vec{W}$. See the figure.

The airfoil has the following characteristics:

$$C_L = 0.15\alpha,$$

$$C_D = 0.015 + 0.015|\alpha|.$$

α is the angle of attack expressed in degrees. The two formulas above are valid only for $-15° < \alpha < 15°$. If α exceeds 15°, the wing stalls and the lift falls to essentially zero.

15.44

Conditions are STP.

Calculate the velocity, W, with which the vehicle moves (after attaining a steady velocity) as a function of the setup angle, ξ. Plot both W and α as a function of ξ.

Prob 15.12 An electric generator, rated at 360 kW at 300 rpm and having 98.7% efficiency at any reasonable speed, produces power proportionally to the square of the number of rpm with which it is driven.

This generator is driven by a windmill that, under given wind conditions, has a torque of 18,000 Nm at 200 rpm, but produces no torque at both 20 and 300 rpm. Assume that the torque varies linearly with the number of rpm between 20 and 200 and between 200 and 300 rpm.

What is the electric power generated?

Prob 15.13 A car is equipped with an electric motor capable of delivering a maximum of 10 kW of mechanical power to its wheels. It is on a horizontal surface. The rolling friction (owing mostly to tire deformation) is 50 N regardless of speed. There is no drag component proportional to the velocity. Frontal area is 2 m^2 and the aerodynamic drag coefficient is $C_D = 0.3$.

Assume calm air at 300 K and at 1 atmosphere. What is the cruising speed of the car at full power?

With the motor uncoupled and a tail-wind of 70 km/h, what is the car's steady state velocity. The drag coefficient is $C_D = 1$ when the wind blows from behind.

Prob 15.14 A building is 300 m tall and 50 m wide. Its C_D is 1. What is the force that the wind exerts on it if it blows at a speed of 10 m/s at a height of 5 m and if its velocity varies with $h^{1/7}$ (h being height above ground).

Prob 15.15 A windmill has a torque versus angular velocity characteristic that can be described by two straight lines passing through the points:

50 rpm, torque = 0;
100 rpm, torque = 1200 Nm;
300 rpm, torque = 0.

1. If the load absorbs power according to $P_{load} = 1000\omega$, what is the power taken up by this load? What is the torque of the windmill? What is the angular velocity , ω?
2. If you can adjust the torque characteristics of the load at will, what is the maximum power that you can extract from the windmill? What is the corresponding angular velocity?

Prob 15.16 A windmill with a swept area of 1000 m^2 operates with 56% efficiency under STP conditions.

At the location of the windmill, there is no wind between 1800 and 0600. At 0600, the wind starts up and its velocity increases linearly with time from zero to a value that causes the 24-h average velocity to be 20 m/s. At 1800, the wind stops abruptly.

What is the maximum energy the windmill can generate in one year?

Prob 15.17 US. Windpower operates the Altamont Wind Farm near Livermore, CA. They report an utilization factor of 15% for their 1990 operation. Utilization factor is the ratio of the energy generated in one year, compared with the maximum a plant would generate if operated constantly at the rated power. Thus, a wind turbine rated at 1 kW would produce an average of 150 W throughout the year. Considering the intermittent nature of the wind, a 15% utilization factor is good. Hydroelectric plants tend to operate with a 50% factor.

1. It is hoped that the cost of the kWh of electricity will be as low as 5 cents. Assuming that the operating costs per year amount to 15% of the total cost of the wind turbine and that the company has to repay the bank at a yearly rate of 12% of the capital borrowed, what cost of the wind turbine cannot be exceeded if the operation is to break even? For your information, the cost of a fossil fuel or a hydroelectric plant runs at about 1000 dollars per installed kW.
2. If, however, the wind turbine actually costs $1000 per installed kW, then, to break even, what is the yearly rate of repayment to the bank?

Prob 15.18 Many swimmers specialize in both free style (crawl) and butterfly. The two strokes use almost the same arm motion, but the crawl uses an alternate stroke whereas the butterfly uses a simultaneous one. The kicks are different but are, essentially, equally effective. Invariably, swimmers go faster using the crawls.

From information obtained in this course, give a first order explanation of why this is so.

Prob 15.19 *Solve equations by trial and error. Use a computer*

An air foil of uniform chord is mounted vertically on a rail so that it can move in a single direction only. It is as if you had cut off a wing of an airplane and stood it up with its longer dimension in the vertical. The situation is similar to the one depicted in the figure of Problem 15.11.

There is no friction in the motion of the air foil. The only drag on the system is that produced by the drag coefficient of the airfoil.

The *setup angle* is the angle between the airfoil reference plane and the normal to the direction of motion. In other words, if the rail is north-south, the air foil faces east when the setup angle is zero and faces north when the set-up angle is 90°.

Consider the case when the set-up angle is zero and the wind blows from the east (wind is abeam). Since the wind generates a lift, the airfoil will move forward. What speed does it attain?

Conditions are STP. Area of the airfoil is 10 m^2. Wind speed is 10 m s^{-1}

The coefficients of lift and drag are given by

$$C_L = 0.6 + 0.066\alpha - 0.001\alpha^2 - 3.8 \times 10^{-5}\alpha^3.$$

$$C_D = 1.2 - 1.1\cos\alpha.$$

Prob 15.20 Consider a vertical rectangular (empty) area with an aspect ratio of 0.5 (taller than wide) facing a steady wind. The lower boundary of this area is 20 m above ground and the higher is 200 mm above ground. The wind velocity at 10 m is 20 m/s and it varies with height according to the $h^{1/7}$ law.

Calculate:

1. the (linear) average velocity of the wind over this area,
2. the cubic mean velocity of the wind over this area,
3. the available wind power density over this area,
4. the mean dynamic pressure over this area.

Assume now that the area is solid and has a C_D of 1.5. Calculate:

5. the mean pressure over this area,
6. the torque exerted on the root of a vertical mast on which the area is mounted.

Prob 15.21 The retarding force, F_D, on a car can be represented by

$$F_D = a_0 + a_1V + a_2V^2.$$

To simplify the math, assume that $a_1 = 0$.

A new electric car is being tested by driving it on a perfectly horizontal road on a windless day. The test consists of driving the vehicle at constant speed and measuring the energy used up from the battery. Exactly 15 kWh of energy is used in each case.

When the car is driven at a constant 100 km/h, the distance covered is 200 km. When the speed is reduced to 60 km/h, the distance is 362.5 km.

If the effective frontal area of the car is 2.0 m^2, what is the coefficient of aerodynamic drag of the vehicle?

Prob 15.22 The army of Lower Slobovia needs an inexpensive platform for mounting a reconnaissance camcorder that can be hoisted to some height between 200 and 300 m. The proposed solution is a kite that consists of a Göttingen 420 air foil with 10 m^2 of area tethered by means of a 300 m long cable. To diminish the radar signature the cable is a long monocrystal fiber having enormous tensile strength so that it is thin enough to be invisible, offers no resistance to airflow and has negligible weight.

In the theater of operation, the wind speed is a steady 15 m/s at an anemometer height of 12 m, blowing from a 67.5° direction. It is known that this speed grows with height exactly according to the 1/7–power law.

The wing loading (i.e., the total weight of the kite per unit area) is 14.9 kg m^{-2}.

The airfoil has the following characteristics:

$$C_L = 0.5 + 0.056\alpha,$$

$$C_D = 0.05 + 0.012|\alpha|,$$

where α is the attack angle in degrees.

The above values for the lift and drag coefficients are valid in the range $-10° < \alpha < 15°$.

The tethering mechanism is such that the airfoil operates with an angle of attack of 0.

The battlefield is essentially at sea level.

The questions are?

1. Assume that the kite is launched by somehow lifting it to an appropriate height above ground. What is the hovering height?
2. What modifications must be made so that the kite can be launched from ground (i.e., from the height of 12 m). No fair changing the wing loading. Qualitative suggestions with a minimum of calculation are acceptable.

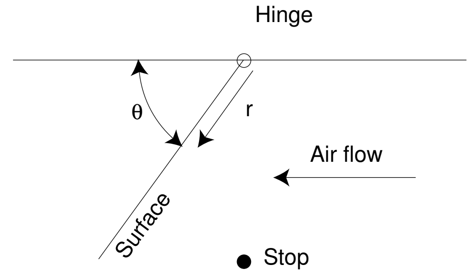
Prob 15.23 A car massing 1000 kg, has an effective frontal area of 2 m^2. It is driven on a windless day on a flat, horizontal highway (sea level) at the steady speed of 110 km/h. When shifted to neutral, the car will, of course, decelerate and in 6.7 seconds its speed is down to 100 km/h.

From these data, estimate (very roughly) the coefficient of aerodynamic drag of the car. What assumptions and/or simplifications did you have to make to reach such estimate?

Is this estimate of C_D an upper or a lower limit? In other words, do you expect the real C_D to be larger or smaller than the one you estimated?

Prob 15.24 In early airplanes, airspeed indicators consisted of a surface exposed to the wind. The surface was attached to a hinge (see figure) and a spring (not shown) torqued the surface so that, in absence of air flow, it would hit a stop and, thus, assume a position with $\theta = 90°$.

The wind caused the surface to move changing θ. This angle, seen by the pilot, was an indication of the air speed.



In the present problem, the surface has dimensions L (parallel to the axis of the hinge) by D (perpendicular to L). $L = 10$ cm, $D = 10$ cm.

The spring exerts a torque

$$\Upsilon_{spring} = \frac{0.1}{\sin\theta} \quad \text{N m.}$$

The coefficient of drag of the surface is 1.28. Air density is 1.29 kg/m^3.

Calculate the angle, θ, for wind velocities, V, of 0, 10, 20, and 50 m/s.

Prob 15.25 An EV (electric vehicle) is tested on a horizontal road. The power, P, delivered by the motors is measured in each run, which consists of a 2 km stretch covered at constant ground velocity, V. Wind velocity, W, may be different in each run.

Here are the test results:

Run	Wind Direction	Wind Speed (m/s) W	Car Speed (km/h) V	Power (kW) P
1	—	0	90	17.3
2	Head wind	10	90	26.6
3	Head wind	20	90	39.1
4	—	0	36	2.1
5	Tail wind	35	90	4.3

How much power is required to drive this car, at 72 km/h, into a 30 m/s head wind?

Prob 15.26 Here are some data you may need:

Quantity	Earth	Mars	Units
Radius	6.366×10^{6}	3.374×10^{6}	m
Density	5,517	4,577	kg/m^3
Surface air pressure	1.00	0.008	atmos.
Air composition	20% O_2, 80% N_2	100% CO_2	
Gravitational constant	6.672×10^{-11}		$N\ m^2 kg^{-2}$

A parachute designed to deliver a 105 kg load to mars is tested on Earth when the air temperature is 298 K and the air pressure is 1.00 atmospheres. It is found that it hits the surface with a speed of 10 m/s.

Assume that mass of the parachute itself is negligible. Assume the drag coefficient of the parachute is independent of the density, pressure, and temperature of the air.

If we want to have a similar parachute deliver the same load to Mars, what must be its area be compared with the area of the test parachute used on Earth?

Prob 15.27 An EV experiences an aerodynamic drag of 320 N when operated at sea level (1 atmosphere) and 30 C.

What is the drag when operated at the same speed at La Paz, Bolivia (4000 m altitude, air pressure 0.6 atmospheres) and at a temperature of -15 C?

Prob 15.28 A trimaran is equipped with a mast on which a flat rigid surface has been installed to act as a sail. This surface is kept normal to the induced wind direction. The boat is 25 km from the shore which is due north of it. A 36 km/h wind, V, blows from south to north. How long will it take to reach the shore if it sails straight down-wind? Ignore any force the wind exerts on the boat except that on the sail.

The area, A, of the sail is 10 m^2.

The coefficient of drag of a flat surface is C_D =1.28.

The air density is $\rho = 1.2$ kg/m^3.

The water exerts a drag force on the trimaran given by

$$F_{water} = 0.5 \times W^2,$$

where, W, is the velocity of the boat relative to the water (there are no ocean currents).

Prob 15.29 Two identical wind turbines are operated at two locations with the following wind characteristics:

Location 1

Percent of time	Wind speed
50	10 m/s
30	20 m/s
20	25 m/s

Location 2

Percent of time	Wind speed
50	15 m/s
50	21 m/s

Which wind turbine generates more energy? What is the ratio of energy generated by the two wind turbines?

Prob 15.30 What is the air density of the planet in Problem 1.22 if the temperature is 450 C and the atmospheric pressure is 0.2 MPa?

Prob 15.31 One may wonder how an apparently weak effect (the reduction of pressure on top of an airfoil caused by the slightly faster flow of air) can lift an airplane.

Consider a Cessna 172 (a small 4-seater). It masses 1200 kg and has a total wing area of 14.5 m^2. In horizontal flight at sea level, what is the ratio of the average air pressure under the wing to the pressure above the wing?

Prob 15.32 A car has the following characteristics:

Mass, m, = 1,200 kg.
Frontal area, A, = 2.2 m^2.
Coefficient of drag, C_D, = 0.33.

The experiment takes place under STP conditions.

When placed on a ramp with a $\theta = 1.7°$ angle, the car (gears in neutral, no brakes) will, of course, start moving and will accelerate to a speed of 1 m/s. This speed is maintained independently of the length of the ramp. In other words, it will reach a **terminal velocity** of 1 m/s.

When a steeper ramp is used ($\theta = 2.2°$), the terminal speed is 3 m/s.

Now place the car on a horizontal surface under no wind conditions. Accelerate the car to 111.60 km/h and set the gears to neutral. The car will coast and start decelerating. After a short time, Δt, the car will have reached the speed of 104.4 km/h.

What is the value of Δt?

Prob 15.33 The observed efficiency of a "gyromill" type wind turbine is given by

$\eta = 0,$ for $\frac{U}{V} \leq 2,$
$\eta = 0.280 \left(\frac{U}{V} - 2\right),$ for $2 \leq \frac{U}{V} \leq 5,$

$\eta = -0.420\frac{U}{V} + 2.940, \quad$ for $\frac{U}{V} > 5$.

The turbine has 2 blades or wings each 30 m long and the radius of the device is 9 m.

When operating at sea level under a uniform 15-m/s wind what power does it deliver to a load whose torque is 50,000 Nm independently of the rotational speed? What is the rotation rate of the turbine (in rpm)?

Prob 15.34 A standard basket ball has a radius of 120 mm and a mass of 560 grams. Its coefficient of drag, C_D, is 0.3 (a wild guess), independently of air speed.

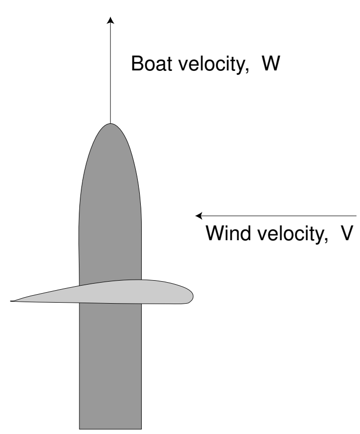
Such a ball is dropped from an airplane flying horizontally at 12 km altitude over the ocean. What is the velocity of the ball at the moment of impact on the water?

Make reasonable assumptions.

Prob 15.35 This is a terrible way to sail a ship, but leads to a simple problem.

In the absence of wind relative to the boat, a boat's engine power, P_{Eng}, of 20,680 W is needed to maintain a speed of 15 knots (1 knot is 1852 meters per hour). The efficiency of the propeller is 80%. Assume that water drag is proportional to the water speed squared.

Under similar conditions, only 45 W are needed to make the boat move at 1 m/s.



This very boat, is now equipped with an 10 m^2 airfoil, mounted vertically and oriented perpendicularly to the boat's axis. See figure.

The coefficient of lift of the airfoil is

$$C_L = (0.05\alpha + 0.5)$$

and is valid for $-10 < \alpha < 10$. In these two equations, α is in degrees.

The airfoil exerts a lift that, it is to be hoped, propels the boat due north when a 15 knot wind blows from the east.

What is the speed of the boat?

Prob 15.36 A sail plane (a motorless glider) is at a 500 m altitude and is allowed to glide down undisturbed. The atmosphere is perfectly still (no winds, no thermals [vertical winds]). Air temperature is 0 C and air pressure is 1 atmosphere.

The wings have a 20 m^2 area and at their lift coefficient is $C_L = 0.5$ and their drag coefficient is $C_D = 0.05$. Assume, to simplify the problem, that the rest of the sail plane (fuselage, empennage, etc.) produces no lift and no drag. The whole machine (with equipment and pilot) masses 600 kg.

Naturally, as the sail plane moves forward, it looses some altitude. The *glide ratio* is defined as the ratio of distance moved forward to the altitude lost.

1. What is the glide ratio of this sail plane?
2. What is the forward speed of the plane?
3. To keep the plane flying as described, a certain amount of power is required. Where does this power come from and how much is it?

Prob 15.37 The drag force on a car can be expressed as power series in v, the velocity of the car (assuming no external wind):

$$F_D = a_0 + a_1 v + a_2 v^2. \tag{75}$$

For simplicity, assume $a_0 = 0$.

A car drives 50 km on a horizontal road (at sea level) at a steady speed of 60 km/h. Careful measurements show that a total of 1.19×10^7 J were used. Next, the car drives another 50 km at a speed of 120 km/h and uses 3.10×10^7 J. The frontal area of the car is 2.0 m^2. What is the coefficient of drag. C_D, of the car?

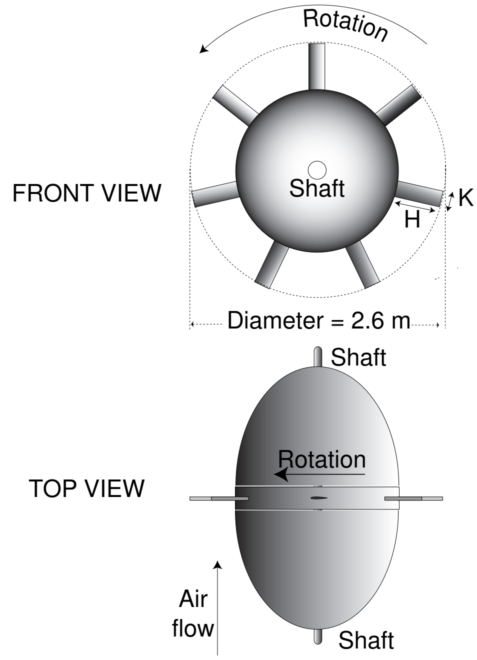
Prob 15.38

Percentage of time	m/s
10	Calm
20	5
40	10
30	15

The wind statistics (over a whole year) at a given site are as shown in the table.

When the wind has a speed of 15 m/s, the wind turbine delivers 750 kW. What is the number of kWh generated in a one year period?

Prob 15.39



Consider a turbine consisting of seven blades each shaped as a NACA W1 symmetric airfoil with a 32 cm chord, K, and sticking out 52 cm, H, above a fairing. These blades have their reference plane aligned with the plane of rotation of the turbine. The diameter is 2.6 m. The device spins at 1050 rpm in such a direction that, if the turbine rotates in calm air, the angle of attack is zero.

To an acceptable approximation, the coefficients of the airfoil are:

$$C_L = 0.08\alpha - 0.0001\alpha^3$$

For $|\alpha| < 11^\circ$:

$$C_D = 0.0062 \exp(0.2|\alpha|)$$

For $11^\circ < |\alpha| < 21^\circ$:

$$C_D = -0.415 + 0.0564|\alpha| - 0.001 * |\alpha|^2.$$

In the formulas above, α is in degrees.

The air stream that drives the turbine flows vertically up in the drawing (it flows parallel to the turbine shaft) and has a density 3 times that of the air at STP. It's velocity is 28.6 m/s.

How much power does the turbine deliver to the shaft?