Chapter 12
Solar Radiation

12.1 The Nature of the Solar Radiation

The sun radiates in all regions of the spectrum, from radio waves to gamma rays. Our eyes are sensitive to less than one octave of this, from 400 to 750 THz (750 to 400 nm), a region known, for obvious reasons, as **visible**. Although narrow, it contains about 45% of all radiated energy. At the distance of one astronomical unit, the power density of the solar radiation is about 1360 W m^{-2}, a value called **solar constant**, which is not really constant—it varies a little throughout the year, being largest in January when the earth is nearest the sun.

The expression **power density** is used to indicate the number of watts per square meter. This is also known as **energy flux**. We will use the expression **spectral power density** to indicate the power density per unit frequency interval or per unit wavelength interval.

Roughly, the distribution of energy over different spectral regions is

Infrared and below(f<400 THz, λ>750 nm)	46.3%
Visible (400 THz<f<750 THz, 400 nm <λ<750 nm)	44.6%
Ultraviolet and above (f>750 THz, λ<400 nm)	9.1%

A much more detailed description of the solar radiation is given in Table 12.1 which shows the fraction, G, of the solar constant associated with frequencies larger than a given value, f. These data, plotted in Figure 10.13 (see the chapter on Production of Hydrogen), correspond to the spectral power density distribution shown in Figure 12.1. For comparison, the spectral power density distribution of black body radiation (6000 K) is plotted for constant wavelength intervals and constant frequency intervals. Notice that these two distributions, although describing the very same radiation, peak at different points of the spectrum. To understand the reason for this apparent paradox, do Problem 12.5.

All of the above refers to radiation outside earth's atmosphere. The power density of solar radiation on the ground is smaller than that in space owing to atmospheric absorption. Radiation of frequencies above 1000 Thz (λ < 300 nm) is absorbed by the upper atmosphere causing photochemical reactions, producing photoionization, and generally heating up the air. However, this part of the spectrum contains only 1.3% of the solar constant. The ozone layer near 25 km altitude absorbs much of it. Ozone is amazingly opaque to ultraviolet. If the atmosphere were a layer of gas of uniform sea level density, it would be 8 km thick. The ozone layer would then measure 2 mm.

Table 12.1
Cumulative Values of Solar Power Density
Fraction of Total Power.
Data from F. S. Johnson.

f (Thz)	G	f (Thz)	G	f (Thz)	G	f (Thz)	G
43	0.9986	176	0.9083	536	0.3180	779	0.0778
50	0.9974	188	0.8940	541	0.3120	789	0.0735
60	0.9951	200	0.8760	545	0.3050	800	0.0690
61	0.9948	214	0.8550	550	0.2980	811	0.0642
63	0.9945	231	0.8290	556	0.2900	822	0.0595
64	0.9941	250	0.7960	561	0.2830	833	0.0553
65	0.9938	273	0.7570	566	0.2760	845	0.0510
67	0.9933	300	0.7090	571	0.2690	857	0.0469
68	0.9929	316	0.6810	577	0.2630	870	0.0427
70	0.9923	333	0.6510	583	0.2560	882	0.0386
71	0.9918	353	0.6170	588	0.2490	896	0.0346
73	0.9913	375	0.5790	594	0.2420	909	0.0308
75	0.9905	400	0.5370	600	0.2350	923	0.0266
77	0.9899	405	0.5270	606	0.2280	938	0.0232
79	0.9891	411	0.5180	612	0.2200	952	0.0233
81	0.9883	417	0.5080	619	0.2130	968	0.0166
83	0.9874	423	0.4980	625	0.2060	984	0.0150
86	0.9863	429	0.4880	632	0.1980	1000	0.0130
88	0.9852	435	0.4780	638	0.1900	1017	0.0106
91	0.9839	441	0.4670	645	0.1820	1034	0.0085
94	0.9824	448	0.4560	652	0.1750	1053	0.0070
97	0.9808	455	0.4450	659	0.1670	1071	0.0059
100	0.9790	462	0.4330	667	0.1590	1091	0.0051
103	0.9772	469	0.4210	674	0.1510	1111	0.0042
107	0.9747	476	0.4090	682	0.1440	1132	0.0035
111	0.9721	484	0.3970	690	0.1370	1154	0.0029
115	0.9690	492	0.3840	698	0.1300	1176	0.0025
120	0.9657	500	0.3720	706	0.1240	1200	0.0021
125	0.9618	504	0.3650	714	0.1170	1224	0.0018
130	0.9571	508	0.3590	723	0.1100	1250	0.0016
136	0.9520	513	0.3520	732	0.1030	1277	0.0014
143	0.9458	517	0.3450	741	0.0970	1304	0.0011
150	0.9387	522	0.3390	750	0.0908	1333	0.0008
158	0.9302	526	0.3320	759	0.0860	1364	0.0006
167	0.9203	531	0.3250	769	0.0819		

12.2

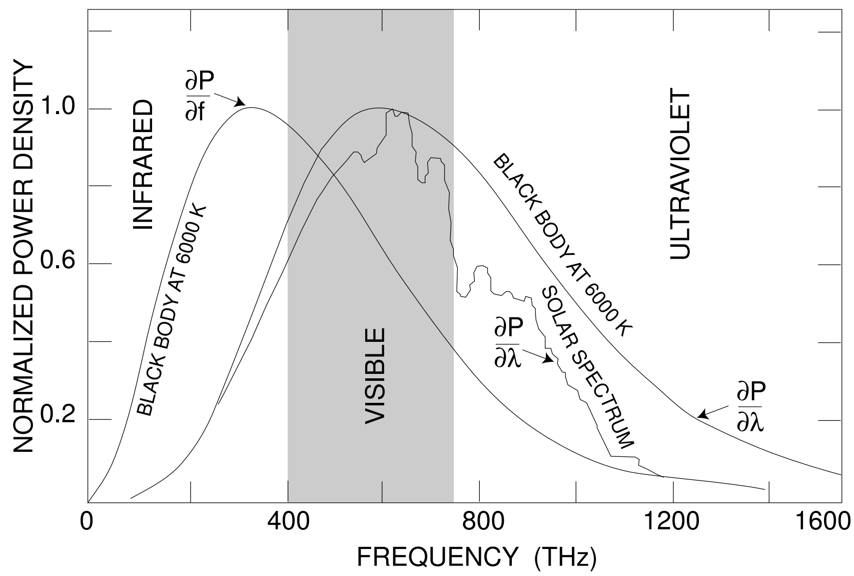

Figure 12.1 The solar power density spectrum compared with that of a black body.

Although solar radiation is generated by several different mechanisms, the bulk of it is of the black body type. The energy per unit volume per unit frequency interval inside a hollow isothermal black body is given by Plank's law:

$$\frac{dW}{df} = \frac{8\pi h}{c^3}\frac{f^3}{\exp(hf/kT) - 1} \qquad \text{J m}^{-3}\text{Hz}^{-1}. \qquad (1)$$

In the preceding expression, W is the energy concentration. The energy flux is equal to the energy concentration times the speed of light (just as a particle flux is equal to the particle concentration times the speed of the particle). Energy flux is, as we stated, the same as power density, P:

$$\frac{dP}{df} \propto \frac{f^3}{\exp(hf/kT) - 1} \qquad \text{W m}^{-2}\text{Hz}^{-1}. \qquad (2)$$

In terms of wavelength,

$$\frac{dP}{d\lambda} \propto \frac{\lambda^{-5}}{\exp(hc/kT\lambda) - 1} \qquad \text{W m}^{-2} \text{ per m of wavelength interval.} \qquad (3)$$

When one tries to match a black body spectrum to that of the sun, one has the choice of picking the black body temperature that best fits the *shape* of the solar spectrum (6000 K) or the temperature that, at one astronomical unit, would produce a power density of 1360 W m^{-2} (5800 K).

12.2 Insolation

12.2.1 Generalities

Insolation[†] is the power density of the solar radiation. In Section 12.1, we saw that the insolation on a surface that faces the sun and is just outside earth's atmosphere is called the solar constant. It has a value of 1360 W m^{-2}.

It is convenient to define a **surface solar constant**—that is, a value of insolation on a surface that, at sea level, faces the vertical sun on a clear day. This "constant" has the convenient value of about 1000 W m^{-2} or "one sun." At other than vertical, owing to the larger air mass through which the rays have to pass, the insolation is correspondingly smaller.

American meteorologists depart from the SI and define a new—and unnecessary—unit called a langley. It is one gram calorie per cm^2 per day. To convert langleys to W m^{-2}, multiply the former by 0.4843.

The insolation depends on:

1, the orientation of the surface relative to the sun, and
2. the transparency of the atmosphere.

Caveat

In the discussion below, we will make a number of simplifications that introduce substantial errors in the results but still describes in general terms the way insolation varies throughout the year and during the day. Some of these errors can, as indicated further on, be easily corrected and those that remain are of little consequence for the planners of solar energy collection systems.

The major sources of errors are:

1. We assume that the length of the period between successive sunrises is constant throughout the year. That is not so—see "Equation of Time" in the Appendix B to this chapter.
2. The time used in our formulas is the "Mean Local Time" and differs from the civil time which refers to the time measured at the center of each time zone.

(continued)

[†] One should not confuse *insolation*, from the Latin *sol* = *sun*, with the word of essentially the same pronunciation, *insulation*, from the Latin *insula* = *island*.

(continued)

Corrections for effects a and b can easily be made by introducing the "Time Offset." (See Appendix A)

3. Our formulas consider only the geometry of the situation. The presence of the atmosphere causes diffraction of the light so that the sun is visible even when it is somewhat below the geometric horizon. This causes the apparent sunrise to be earlier than the geometric one and the apparent sunset to be later. This effect can be somewhat corrected by using a solar zenith angle at sunrise and sunset of 90.833° instead of the geometric 90°.

However, it must be remembered that the refraction correction is latitude dependent and becomes much larger near the polar regions. Much more detailed information on the position of the sun can be found at www.srrb.noaa.gov/highlights/sunrise .

4. The insolation data assume perfectly transparent atmosphere. Meteorological conditions do, of course, alter in a major way the amount of useful sun light.

In this part of this book, the position of the sun is characterized by the **zenith angle**, χ (the angle between the local vertical and the line from the observer to the sun), and by the azimuth, ξ, measured clockwise from the north. This is a **topocentric** system—the observer is at the origin of the coordinates. In Appendix B, we will use two different points of view: a **geocentric** system (origin at the center of earth) and a **heliocentric** system (origin at the center of the sun). In our topocentric system, both χ and ξ are functions of:

1. the local time of day, t,[†]
2. the day of the year, d, and
3. the latitude of the observer, λ.

Observe that the time, t, in our formulas is not the time shown in your watch. These times differ by the **time offset** which has two components, one related to the difference in longitude of the place of interest from that of the center of the time zone, and one owing to the **equation of time. EOT** (see appendix). To get a better feeling about these times, do Problem 14.24. t in our formulas is exactly 12:00 when the *mean* sun crosses the local meridian—that is, when the solar zenith angle is at a minimum. The real

[†] For comments on time measurement, please refer to Appendix A to this chapter.

sun is in general either ahead or behind the mean sun by an amount called the EOT.

The time of day is represented by the **hour angle**, α, an usage borrowed from astronomers who in the past worked mostly at night and thus preferred to count a new day from noon rather than from midnight. They define the hour angle as

$$\alpha \equiv \tfrac{360}{24}(t-12) \qquad \text{degrees (t in hours, 24-h clock)} \tag{4}$$

$$\alpha \equiv \tfrac{2\pi}{86400}(t-43200) \qquad \text{radians (t in seconds, 24-h clock)} \tag{4a}$$

The day of the year or "season" is represented by the solar declination, δ—by the latitude of the sun.

The solar declination can be found, for any day of a given year, in the **Nautical Almanac** or by consulting the NOAA or the Naval Observatory website. It also can be estimated with sufficient precision for our purposes by the expression:

$$\delta = 23.44 \sin\left[360\left(\frac{d-80}{365.25}\right)\right] \qquad \text{degrees,} \tag{5}$$

where d is the day number.

The solar zenith angle and the solar azimuth are given by

$$\cos\chi = \sin\delta\sin\lambda + \cos\delta\cos\lambda\cos\alpha \tag{6}$$

and

$$\tan\xi = \frac{\sin\alpha}{\sin\lambda\cos\alpha - \cos\lambda\tan\delta}, \tag{7}$$

where λ is the latitude of the observer.

To find ξ, we have to take $\arctan(\tan\xi)$. Notice, however, that $\arctan(\tan\xi)$ is not necessarily equal to ξ. Consider, for instance, the angle 240° whose tangent is 1.732. A calculator or a computer will tell you that $\arctan 1.732 = 60°$ because such devices yield the **principal value** of arctanξ which, by definition, lies in the range from −90° to 90°. The following rule must be observed when obtaining ξ from Equation 7:

Sign(α)	Sign(tanξ)	ξ
+	+	180° +arctan(tanξ)
+	−	360°+arctan(tanξ)
−	+	arctan(tanξ)
−	−	180°+arctan(tanξ)

12.6

An alternative formula for determining the solar azimuth is

$$\cos(180^\circ - \xi) = -\frac{\sin\lambda\cos\chi - \sin\delta}{\cos\lambda\sin\chi}.$$

At both sunrise and sunset, $\chi = 90^{\circ\dagger}$; thus, $\cos\chi = 0$. From Equation 6,

$$\cos\alpha_R = \cos\alpha_S = -\tan\delta\tan\lambda \tag{8}$$

where $\alpha_{R,S}$ is the hour angle at either sunrise or sunset. The hour angle α_R, at sunrise, is negative and α_S, at sunset, is positive.

$$\alpha_R = -\alpha_S. \tag{9}$$

12.2.2 Insolation on a Sun-Tracking Surface

If a flat surface continuously faces the sun, the daily average insolation is

$$<P>= \frac{1}{T}\int_{t_R}^{t_S} P_S dt \quad \text{W m}^{-2}, \tag{10}$$

where t_R and t_S are, respectively, the times of sunrise and sunset, T is the length of the day (24 hours), and P_S is the solar power density, which, of course, depends on the time of day and on meteorological conditions. Assuming (unrealistically) that P_S is a constant from sunrise to sunset, the average insolation, $<P>$, in terms of the hour angle, is

$$<P>= \frac{1}{\pi}\alpha_S P_S \quad \text{W m}^{-2}. \tag{11}$$

At the equinoxes, $\delta = 0$, and, consequently, $\alpha_S = \pi/2$, and

$$<P>= \tfrac{1}{2}P_S \approx 500 \;\; \text{W m}^{-2} \approx 43.2 \;\; \text{MJ m}^{-2}\text{day}^{-1}. \tag{12}$$

12.2.3 Insolation on a Stationary Surface

Instantaneous insolation on surfaces with elevation, ϵ, azimuth, ζ, is

$$P = P_S[\cos\epsilon\cos\chi + \sin\epsilon\sin\chi\cos(\xi - \zeta)]. \tag{13}$$

Care must be exercised in using Equation 13. The elevation angle, ϵ, is always taken as positive. See Figure 12.2. It is important to check if the

† This will yield the geometric sunrise. As explained before, to correct for atmospheric refraction, χ at sunrise and sunset is taken as 90.833°.

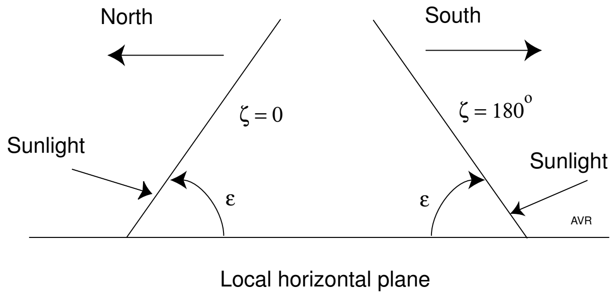

Figure 12.2 The two surfaces above have the same elevation but different azimuths.

sun is shining on the front or the back of the surface. The latter condition would result in a negative sign in the second term inside the brackets. Negative values are an indication that the surface is in its own shadow and that the insolation is zero.

The daily average insolation is

$$<P>= \frac{1}{T}\int_{t_R}^{t_S} Pdt = \frac{1}{2\pi}\int_{\alpha_R}^{\alpha_S} Pd\alpha. \tag{14}$$

For the general case, the preceding integral must be evaluated numerically. Figures 12.3 through 12.5 show some of the results. Figure 12.3 shows the insolation on south-facing surfaces located at a latitude of 40° north and with various elevation angles, all as a function of solar declination. A horizontal surface ($\epsilon = 0$) receives a lot of sun light during the summer ($\delta = +23°$). At the height of summer, it receives more light than at the equator where the average (normalized) insolation is, by definition, 1, independently of the season. As the seasons move on, the insolation diminishes and, in winter, it is less than 40% of that in summer.

A vertical surface facing the equator receives more insolation in winter than in summer. There is an optimum elevation that yields maximum year around insolation and, incidentally, minimum seasonal fluctuation. For the latitude of Figure 12.3 (40°), the optimum elevation is 42°—that is, 2° more than the latitude.

The difference between the optimum elevation and the latitude is plotted, as a function of latitude, in Figure 12.4.

The latitude has small influence on the annual insolation for a surface at optimum elevation angle, as can be seen from Figure 12.5. At 67°, the highest latitude at which the sun comes up every day, the yearly insolation is still well above 80% of that at the equator. This assumes an atmosphere whose transparency does not depend on season and on elevation angles. Actually, the farther north, the larger the atmospheric absorption owing to the smaller average solar elevation angle.

12.8

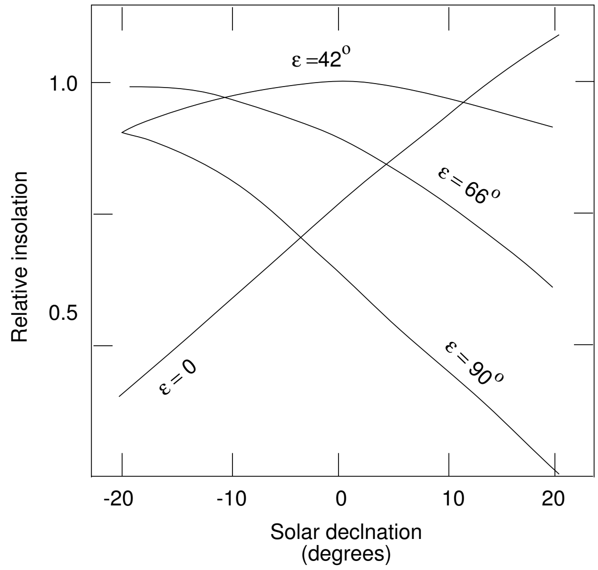

Figure 12.3 Relative insolation of surfaces with various elevations at a latitude of 40°. Each curve has been normalized by comparing the insolation with that on a horizontal surface at the equator.

On the other hand, in many equatorial regions, such as the Amazon valley, the cloud cover is so frequent that the average insolation is only some 60% of that under ideal meteorological conditions.

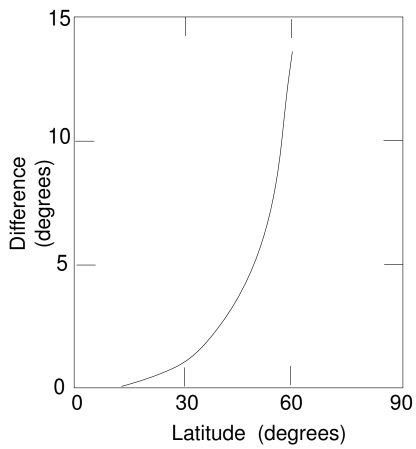

Figure 12.4 Difference between the optimum elevation angle of a solar collector and its latitude.

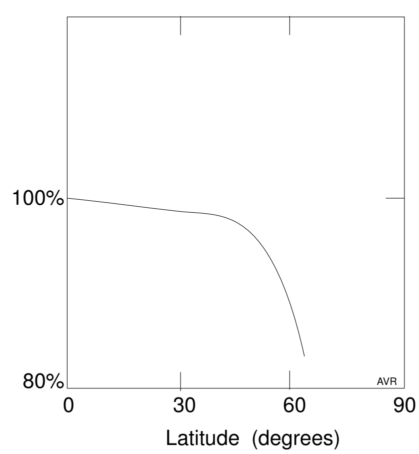

Figure 12.5 Effect of latitude on the annual insolation of a solar collector at optimum elevation.

Table 12.2
Estimated Insolation
South-Facing Surface
Elevation Equal to Latitude

CITY	AVERAGE INSOLATION W/m^2
Bangor, ME	172
Boston, MA	177
Buffalo, NY	161
Concord, NH,	171
Hartford, CT	149
Honolulu, HI	230
Los Angeles, CA	248
Newark, NJ	186
New York, NY	172
Philadelphia, PA	185
Phoenix, AZ	285
San Francisco, CA	246
Tucson, AZ	286

It can be seen that the yearly average insolation depends on the orientation of the surface, the latitude and the prevailing meteorological conditions.

Table 12.2 shows the yearly average insolation at different cities of the USA for south-facing surfaces with an elevation equal to the local latitude. The Table was adapted from one published in IEEE SPECTRUM, October 1996, page 53 whose source was "Optimal BIPV Applications," Kiss and Co., Architects, November 1995.

Clearly, these estimated insolation values vary from year to year owing to the variability of the cloud cover.

12.2.4 Horizontal Surfaces

For horizontal surfaces ($\epsilon = 0$), Equation 13 reduces to

$$P = P_S \cos\chi. \tag{15}$$

Consequently,

$$\begin{aligned} <P> &= \frac{1}{2\pi}\int_{\alpha_R}^{\alpha_S} P_S \cos\chi d\alpha \\ &= \frac{P_S}{2\pi}[\sin\delta\sin\lambda(\alpha_S - \alpha_R) + \cos\delta\cos\lambda(\sin\alpha_S - \sin\alpha_R)] \\ &= \frac{P_S}{2\pi}\cos\delta\cos\lambda(2\sin\alpha_S + 2\alpha_S\tan\delta\tan\lambda) \\ &= \frac{P_S}{\pi}\cos\delta\cos\lambda(\sin\alpha_S - \alpha_S\cos\alpha_S). \end{aligned} \tag{16}$$

At the equinoxes, $\delta = 0$, $\alpha_S = \pi/2$, therefore

$$<P>= \frac{P_S}{\pi} \cos \lambda. \tag{17}$$

At the equator, regardless of δ, $\alpha_S = \pi/2$, therefore

$$<P>= \frac{P_S}{\pi} \cos \delta. \tag{18}$$

12.3 Solar Collectors

Methods for collecting solar energy for the production of either heat or electricity include:

1. appropriate architecture,
2. flat collectors,
3. evacuated tubes,
4. concentrators, and
5. solar ponds.

12.3.1 Solar Architecture

Proper architecture is an important energy saving factor. Among others, the following points must be observed:

12.3.1.1 Exposure Control

Building orientation must conform to local insolation conditions. To provide ambient heating, extensive use can be made of equatorward facing windows protected by overhangs to shield the sun in the summer. Reduction or elimination of poleward facing windows will diminish heat losses. Shrubs and trees can be useful. Deciduous trees can provide shade in the summer while allowing insolation in the winter.

12.3.1.2 Heat Storage

Structures exposed to the sun can store heat. This may be useful even in summer—the heat stored may be used to pump cool air by setting up convection currents.

Roof ponds can contribute to both heating and cooling. Any part of the building (walls, floor, roof, and ceiling) can be used for heat storage.

Figure 12.6 shows an elaborate wall (proposed by Concept Construction, Ltd., Saskatoon, Saskatchewan) that acts as a heat collector. It consists of a 25-cm thick concrete wall in front of which a glass pane has been installed leaving a 5-cm air space.

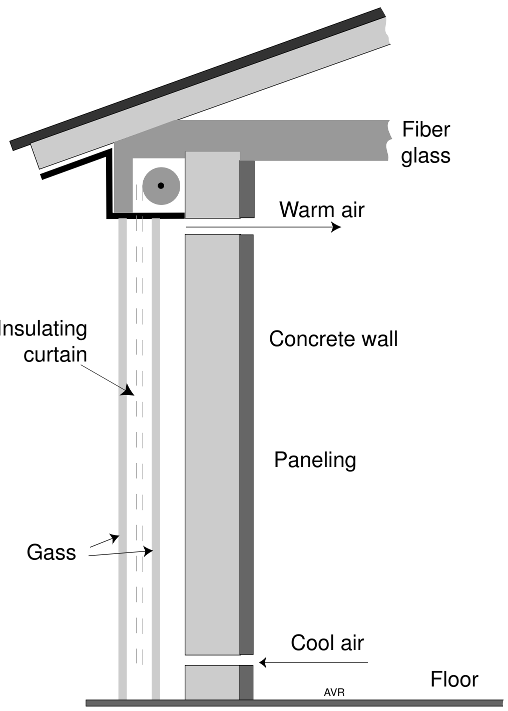

Figure 12.6 Heat storing wall. (Concept Construction, Ltd.)

During the summer, the warmed air is vented outside and the resulting circulation causes cooler air from the poleward face of the house to be taken in.

Instead of a concrete wall, the heat-storing structure can be a large stack of "soda" cans full of water (or, for that matter, full of soda). This takes advantage of the high heat capacity of water. The glazing is placed in contact with the cans so as to force the air to seep through the stack, effectively exchanging heat with it. (You may as well paint the cans black.)

12.3.1.3 Circulation

Heat transfer can be controlled by natural circulation set up in a building by convection currents adjusted by vents. Circulation is important from the health point of view—attempts to save energy by sealing houses may cause an increase in the concentration of radon that emanates from the ground in some places but that is normally dissipated by leakages. Other undesirable gases accumulate, one being water vapor leading to excessive moisture. In addition, noxious chemicals must be vented. This is particularly true of formaldehyde used in varnishes and carpets.

To reduce heat losses associated with air renewal, air-to-air heat exchangers or **recuperators** are used. In this manner, the outflowing air preheats (or pre-cools) the incoming fresh air. About 70% of the heat can be recovered.

12.3.1.4 Insulation

The heat power, P, conducted by a given material is proportional to the heat conductivity, λ, the area, A, and the temperature gradient, dT/dx:

$$P = \lambda A dT/dx. \tag{19}$$

In the S.I., the unit of heat conductivity is $\mathrm{WK^{-1}m^{-1}}$. Many different nonmetric units are used in the USA. Commonly, insulating materials have their conductivity expressed in BTU $\mathrm{hr^{-1}}$ $\mathrm{ft^{-2}}$ $\mathrm{(F/inch)^{-1}}$, a unit that is a good example of how to complicate simple things. To convert from this unit to the S.I., multiply it by 0.144.

Under constant temperature gradient, $dT/dx = \Delta T/\Delta x$ where ΔT is the temperature difference across a material Δx units thick.

One has

$$P = \frac{\lambda A \Delta T}{\Delta x} = \frac{A\Delta T}{R}, \tag{20}$$

re

$$R \equiv \Delta x/\lambda. \tag{21}$$

R has units of $\mathrm{m^2K\ W^{-1}}$ or, in the USA, hr $\mathrm{ft^2F/BTU}$. Again, to convert from the American to the S.I., multiply the former by 0.178.

Insulating materials are rated by their R-values. Fiberglass insulation, 8 cm thick, for instance, is rated R-11 (in the American system).

Consider a house with inside temperature of 20 C and an attic temperature of 0 C. The ceiling has an area of 100 $\mathrm{m^2}$ and is insulated with R-11 material. How much heat is lost through the attic?

In the S.I., R-11 corresponds to $11 \times 0.178 = 2\ \mathrm{m^2\ K\ W^{-1}}$. Thus, the heat loss under the assumed 20 K temperature difference is

$$P = \frac{100 \times 20}{2} = 1000 \quad W. \tag{22}$$

Thus, if the only heat losses were through the ceiling, it would take little energy to keep a house reasonably warm. There are, of course, large losses through walls and, especially through windows. A fireplace is a particularly lossy device. If the chimney is left open, warm air from the house is rapidly syphoned out. If a metallic damper is used to stop the convection current, then substantial heat is conducted through it.

12.3.2 Flat Collectors

Flat collectors work with both direct and diffused light. They provide low temperature heat (less than 70 C) useful for ambient heating, domestic hot water systems, and swimming pools. This type of collector is affected by weather and its efficiency decreases if large temperature rises are demanded.

For swimming pools in the summer, when only a small temperature increase is needed, flat collectors can be over 90% efficient. It is necessary to operate them so that large volumes of water are only slightly heated rather than heating small amounts of water to a high temperature and then mixing them into the pool.

Simple collectors are black plastic hoses exposed to the sun. More elaborate collectors use both front and back insulation to reduce heat losses.

Collectors may heat water directly or may use an intermediate heat transfer fluid.

Figure 12.7 shows a cross-section through a typical flat collector. Light and inexpensive aluminum is used extensively; however, it tends to be corroded by water. Copper is best suited for pipes. If an intermediate heat exchange fluid is employed, aluminum extrusions that include the channels for the liquid are preferred.

Some panels use a thin copper sleeve inserted into the aluminum tubing.

Panels can be black-anodized or painted. There is some question about the lifetime of paints exposed to solar ultraviolet radiation.

The front insulation can be glass or plastic. The former is fragile but the latter does not withstand ultraviolet well. To avoid heat losses through the back of the panel, insulation such as fiberglass mats or polyurethane foam is used. The latter imparts good rigidity to the panel allowing a reduction in the mass of the material employed.

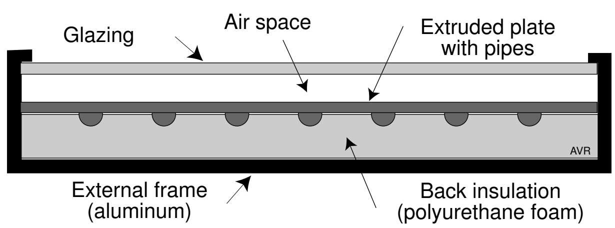

Figure 12.7 Cross-section through a typical flat collector.

12.14

12.3.3 Evacuated Tubes

This type of collector consists of two concentric cylinders, the outer one of glass and the inner, a pipe through which the liquid flows. They bear a superficial resemblance to fluorescent lamps. A vacuum is established between the two cylinders, reducing the convection heat losses.

Evacuated tubes are nondirectional and can heat liquids to some 80 C. They are usually employed in arrays with spacing equal to the diameter of the outer tube. It is customary to place a reflecting surface behind the array.

12.3.4 Concentrators

Concentrators can be of the non-imaging or of the focusing type. Either can be line concentrators (2-D) or point concentrators (3-D).

A solar collector consists of a concentrator and a receiver. The concentrator may be of the refracting (lens) or of the reflecting (mirror) type. The receiver may be thermal or photovoltaic.

Two important parameters describe the collector performance:

1. the concentration, C, and
2. the acceptance angle, θ.

The concentration can be defined as either the ratio of the aperture area to the receiver area, or as the ratio of the power density at the receiver to that at the aperture. These definitions are not equivalent; the latter is preferable.

The acceptance angle is the angle through which the system can be misaimed without (greatly) affecting the power at the receiver (see Figure 12.10).

There is a theoretical relationship between the concentration and the acceptance angle for the ideal case:

$$C_{ideal} = (\sin\theta)^{-1} \qquad \text{for a 2-D concentrator,} \tag{23}$$

$$C_{ideal} = (\sin\theta)^{-2} \qquad \text{for a 3-D concentrator.} \tag{24}$$

It is instructive to calculate the maximum temperature that a receiver can attain as a function of concentration. An ideal receiver will work in a vacuum (no convection losses) and will be perfectly insulated (no conduction losses). Nevertheless, radiation losses are unavoidable. They will amount to

$$P_r = \sigma\epsilon T^4 \qquad \mathrm{Wm^{-2}}, \tag{25}$$

where ϵ is the emissivity (taken as unity)† and σ is the Stefan-Boltzmann

† It is, of course, desirable to have a low emissivity at the temperature at which the receiver operates and a high absorptivity at the region of the spectrum dominated by the incident sunlight.

constant (5.67×10^{-8} W m^{-2}K^{-4}). See Chapter 6.

The power density at the receiver (assuming no atmospheric losses) is

$$P_{in} = 1360\ C \qquad \mathrm{Wm}^{-2}. \tag{26}$$

In equilibrium, $P_r = P_{in}$,

$$\sigma T^4 = 1360\ C \tag{27}$$

or

$$T = (2.4 \times 10^{10} C)^{1/4} = 394\ C^{1/4}. \tag{28}$$

With unity concentration (flat plate collector), $T = 394$ K or 120 C.

When the concentration is raised to 1000, the maximum temperature theoretically attainable is 2,200 K. Were it possible to construct a collector with a concentration of 1 million, the formula would predict a receiver temperature of 12,400 K. This would violate the second law of thermodynamics because heat would be flowing unaided from the cooler sun (6000 K) to the hotter receiver. Clearly, the upper bound of the receiver temperature must be 6000 K.

One can arrive at the same conclusion by considering the expression for C_{ideal}. The solar angular radius, as seen from earth, is $0.25°$. Thus, the minimum accept, and this leads to a $C_{max} = 52,000$ for a 3-D collector and 230 for a 2-D collector as calculated from Equations 23 and 24. A concentration of 52,000 corresponds to a T_{max} of 5900 K (Equation 28), just about right.

Numerous reasons cause the concentration (in terms of power densities) to be less than ideal:

reflector shape and alinement errors,
less than perfect reflector surface reflectivity,
tracking errors,
atmospheric scatter, and
atmospheric absorption.

12.3.4.1 Holographic Plates

Just as a flat plastic sheet with appropriate grooves (a **Fresnel** lens) can concentrate light, a holographic plate can be fashioned to do the same. The advantage of the holographic approach is that the plate can simultaneously diffract and disperse, creating a rain-bow of concentrated light with each region of the spectrum directed to a collector optimized for that particular color range. This is a significant advantage when using photo-voltaic cells. This technology, which looks promising, is discussed in Chapter 14.

12.16

12.3.4.2 Nonimaging Concentrators

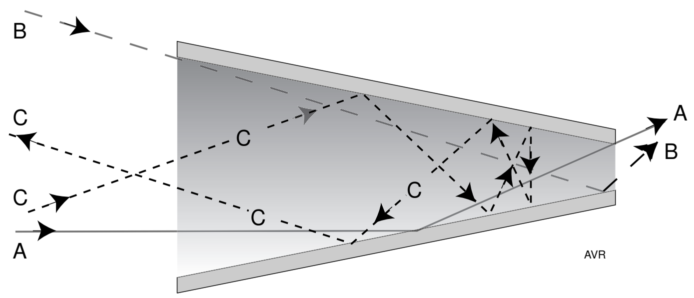

Figure 12.8 Ray paths in a conical concentrator.

The simplest nonimaging concentrator is a cone. In a properly designed cone, all rays parallel to its axis will be reflected into the exit aperture. See ray A in Figure 12.8. The acceptance angle, however, is small. Ray B makes it through the exit, but ray C, although parallel to B, does not—it bounces around a number of times and finally returns to the entrance.

The performance of nonimaging concentrators is improved by the use of a **compound parabolic concentrator**, or CPC.

This device consists of parabolic surfaces as shown in Figure 12.9 (from Welford and Winston). Notice that the section is not that of a truncated parabola but rather, that of two independent parabolas, mirror images of one another. The CPCs in Figure 12.9 have identical exit apertures. The largest has the biggest collecting area and consequently the largest concentration; the acceptance angle, however, is small compared with that of the CPCs with smaller concentration.

The area of the reflecting surfaces of a CPC is much larger than that of a focusing paraboloid of the same concentration. This makes CPCs heavy, expensive, and difficult to mount.

Ideally, a plot of the normalized light power density that leaves the exit aperture as a function of the **aiming angle**, Θ, should be a rectangle: unity for $\Theta < \Theta_i$ and zero for $\Theta > \Theta_i$. Here, Θ_i is the acceptance angle. This ideal characteristic is poorly approached by a conical concentrator, but reasonably well achieved by a CPC. See Figure 12.10.

2-D non-imaging collectors with concentrations up to 2 or 3 need not track the sun. Devices with larger concentrations require a once a month re-aiming to compensate for variations in solar declination.

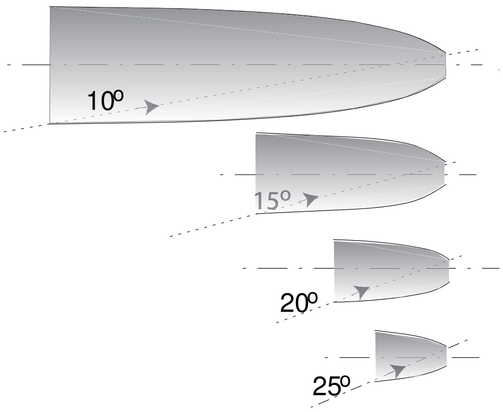

Figure 12.9 For the same exit aperture, the larger the entrance aperture, the smaller the acceptance angle.

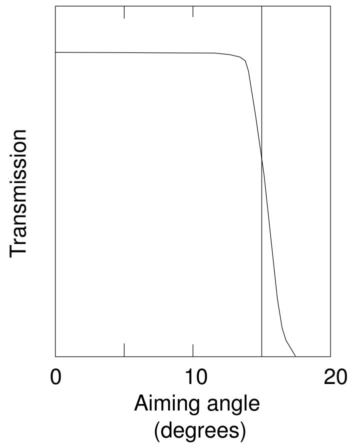

Figure 12.10 A CPC approaches the ideal transmission versus aiming angle characteristics.

12.4 Some Solar Plant Configurations

12.4.1 High Temperature Solar Heat Engine

A straightforward method of generating electricity from solar energy is to use concentrators to produce high temperatures that can drive either a Stirling or a Rankine (steam) engine.

Southern California Edison Company had a 10 MW facility in Barstow, in the Mojave Desert. It cost \$140 million—that is, \$14,000/kW. Since fossil-fueled plants may cost around \$1000/kW, it can be seen that this particular solar thermal installation can be justified only as a development tool. Since the "fuel" is free, it is important to determine the plant life, and the operation and maintenance cost so as to be able to compare the cost of electricity over the long run with that from traditional sources.

The plant operated in the manner indicated schematically in Figure 12.11. The collecting area was roughly elliptical in shape and covered some 300,000 m^2 (30 hectares). Insolation, averaged over 24 hours, is probably around 400 W/m^2 for sun-tracking collectors, leading to an efficiency of the order of 8%.

The collector consisted of 1818 sun-tracking flat mirrors forming a gigantic heliostat capable of focusing the sun's energy on a boiler.

The boiler was a cylinder 7 m in diameter and 14 m in height. It was operated at 788 K (516 C) and 10.7 MPa (105 atmospheres).

A thermal storage system was incorporated having a 100 GJ (electric) capacity permitting the plant to deliver some 7 MW for 4 hours during nighttime.

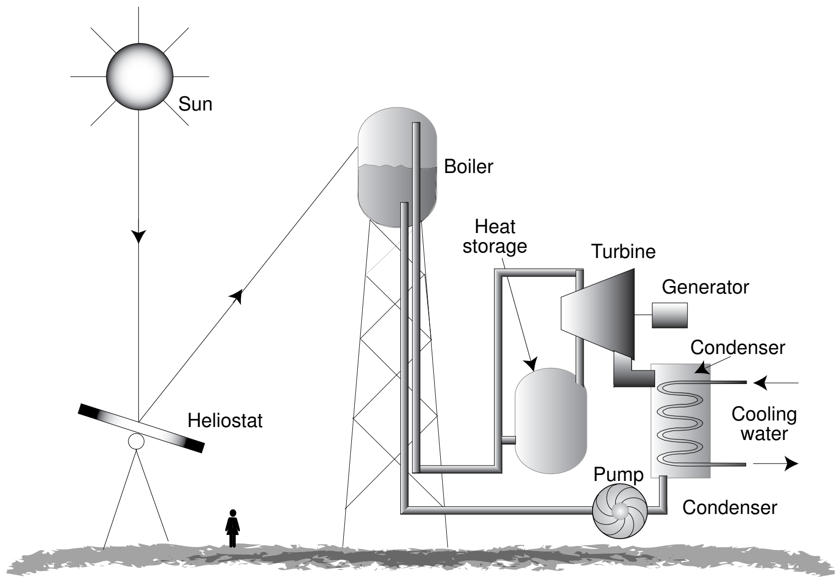

Figure 12.11 Solar One power plant of the Southern California Edison Co., in the Mojave Desert.

Solar One was decommissioned and an update installation, Solar Two, took its place starting operation in July 1996. The new plant cost $48.5 million but inherited considerable assets from the previous effort (among other, the many heliostats) so that its true investment cost is hard to estimate.

Solar Two has the same 10 MW rating of Solar One. The main difference is the use of an intermediate working fluid—a $NaNO_3/KNO_3$ mix containing 40% of the potassium salt—that is heated in the solar tower and transfers its energy to the turbine operating steam by means of a heat exchanger. The salt leaves the tower at 565 C and, after delivering part of its heat energy to the steam, returns to the tower at 288 C.[†]

The salt mixture is somewhat corrosive requiring low grade stainless steel in pipes and containers. At the operating temperature the mixture is quite stable and has a low vapor pressure.

[†] Both sodium nitrate and potassium nitrate go by the common name of **niter** or **saltpeter**. Sodium nitrate (a common fertilizer and oxidant) melts at 306.8 C and potassium nitrate (used in the manufacture of black powder) melts at 334 C. Usually, alloys and mixtures melt at a lower temperature than their constituents. **Eutectic** mixture is the one with the lowest melting point. Solar Two uses less potassium than the eutectic, because the potassium salt is more expensive than the sodium one. The mixture used in the plant melts at 288 C.

The mirrors are made of a sandwich of two glass panes with a silver layer between them. This protects the reflecting layer from corrosion. When not in use, the mirrors are placed in a horizontal position to protect them from the destructive action of wind storms. This also reduces abrasion from wind carried sand.

12.4.2 Solar Chimney

A circular tent with 121 m radius made mostly of plastic material (and, partially, of glass), was built in Manzanares, Spain. The height of the tent is 2 m at the circumference and 8 m at the center where a 194-m tall chimney has been erected.

The air under the tent is heated by the greenhouse effect and rises through the 10-m diameter chimney driving a 50 kW turbine.

The installation, owned by the Bundesministerium für Forschung und Technologie, Bonn, Germany, was built by Schlaich Bergermann und Partner. It operated from 1989 through 1996 and served to demonstrate the principle.

In 2002, the Australian Minister for Industry, Tourism and Resources gave its support to the firm EnviroMission to work on a much larger plant of the Manzanares type. The plant would generate at peak 200 MW of electricity and would cost $800 million. This corresponds to $4000/kW a reasonable amount for a development project (Typically, electric generating plants cost about $1000/kW). The tower is supposed to generate 650 GW-hours per year (2.3×10^{15} J/year). This represents an optimistic 36% utilization factor.

The proposed plant would have a collecting tent of 7 km diameter—that is, a collecting area of some 38×10^6 m^2. At a peak insolation of 1000 W/m^2, the efficiency would be about 0.5%.

Since the efficiency of the system depends on the height of the chimney, the design requires a 1000-m high structure. Such a tall structure will certainly be a challenge to civil engineers because (among other factors) it would be subject to enormous wind stresses. The tallest existing tower (excluding radio towers) is the 553-m Canada's National Tower in Toronto.

12.4.3 Solar Ponds

The OTEC principle discussed in Chapter 4 can be used to generate electricity from water heated by the sun. Surprisingly high temperatures can be obtained in some ponds.

In shallow ponds with a dark colored bottom, the deep layers of water are warmed up and rise to the surface owing to their lower density. This causes mixing that tends to destroy any temperature gradient. In such ponds the temperature rise is modest because most heat is lost through the surface by the evaporation of water.

The solution is, of course, to cover the pond with an impermeable light-transmitting heat-insulating layer such as it is done in swimming pools with plastic covers.

It is interesting to observe that the insulating layer can be the water itself. If a vertical salinity gradient is created in the pond, so that the deeper layers contain more salt and become correspondingly denser, it is possible to impede convection and achieve bottom temperatures as high as 80 C.

Working against a 20 C cold sink, the Carnot efficiency of an OTEC would be 20% and practical efficiencies of 10% do not seem impossible.

Difficulties involve:

1. mixing owing to wind action and other factors,
2. development of turbidity owing to collected dirt and the growth of microorganisms.

It may be possible to overcome such difficulties by using **gel ponds** in which the water is covered by a polymer gel sufficiently viscous to impede convection. Such gel must be

1. highly transparent to sunlight,
2. stable under ultraviolet radiation,
3. stable at the operating temperature,
4. insoluble in water,
5. nonbiodegradable,
6. nontoxic,
7, less dense than the saline solution, and
8. inexpensive.

E.S. Wilkins and his colleagues at the University of New Mexico (Albuquerque) claim to have developed such a gel.

To keep the surface clean, a thin layer of water runs on top of the gel sweeping away dirt and debris.

Appendix A (The Measurement of Time)

The Duration of an Hour†

How long is an hour?

In Roman times, the hour was defined as 1/12 of the time period between sunrise and sunset. Since this interval varies with seasons, the "hour" was longer in the summer than in the winter.

At the latitude of Rome, about 42° N, one hour would last anywhere between 44.7 modern minutes (in the end of December) and 75.3 (in mid-June). This variability was a major problem for clockmakers who had to invent complicated mechanisms to gradually change the clock speed according to the season of year. See *On Architecture* by Vitruvius (Marcus Vitruvius Pollio, a book still being sold).

Much of the variability is eliminated by defining the hour as 1/24 of the interval between two consecutive noons, i.e., two consecutive solar crossings of the local meridian. Unfortunately, this also leads to an hour whose length varies throughout the year, albeit much less than the Roman one. See detailed explanation in Appendix B to this chapter. The obvious solution is to define a **mean solar hour** as the average value of the **solar hour** taken over one year interval. But again, owing to the very slow changes in astronomical constants (eccentricity, semi major axis, argument of perihelion, etc.), this definition of an hour will not be constant over long periods of time. The final solution is to define arbitrarily an **ephemeris hour** referred to an atomic clock. At present, the ephemeris hour is very close to the mean solar hour.

Astronomy and chronology being ancient sciences have inherited ancient notions and ancient terminology. The division of the hour into minutes and seconds is one example.

Using the Babylonian sexagesimal system, the hour was divided into "minute" parts (*pars minuta prima* or first minute part or simply "minutes") and then again into *pars minuta secunda* or simply, "seconds." The latter is the official unit of time for scientific purposes, and has a value of 1/86,400 of a mean solar day.

The times obtained from the approximate formulas in this book are the mean solar times and may differ by as much as ±15 minutes from the true solar times.

Time Zones

The local mean solar time is not a convenient measure of time for every day use because it depends on the longitude of the observer. It varies by

† Fore more information on the history of the hour, read Dohrn-van Rossum. See Further Reading at the end of this chapter.

1 hour for every 15° of longitude. This means, of course, that the time in San Francisco is not the same as in Sacramento. The use of time zones, 1 hour or 15° wide, circumvents this difficulty. In each zone, the time is the same regardless of the position of the observer. At the zone boundaries, the time changes abruptly by 1 hour. The center meridian of any time zone is a multiple of 15°; the first zone is squarely centered on the zeroth meridian, that of Greenwich and the time there is called **Greenwich mean time, GMT** (or, to astronomers, **universal time, UT**). The zone time is called **standard time** (such as, for instance, PST, for Pacific Standard Time, the −8 time zone centered at 120° W).

Time Offset

The true solar time, t_{true}, at any given longitude, L, can be found from

$$t_{true} = t_{local\ mean} + t_{offset}, \tag{29}$$

where t_{true} and t_{local} are expressed in hours and minutes but t_{offset} is in minutes only,

$$t_{offset} = EOT - 4L + 60t_{zone} \quad \text{minutes}, \tag{30}$$

where EOT is the equation of time (in minutes) discussed in Appendix B, L is the longitude in degrees (east > 0, west< 0), and t_{zone} is the number of hours of the local time zone referred to the UT (east > 0, west < 0).

A simple example:

What is the true solar time on February 20, at Palo Alto, CA (125° W) when the local mean time is 12:00 (noon)?

The EOT for February 20 (scaled from Figure 12.11) is +14 minutes. The time zone of Palo Alto is Pacific Standard Time, i.e., it is -8 h. We have

$$t_{offset} = 14 - 4 \times (-125) + 60(-8) = 34 \quad \text{minutes} \tag{31}$$

$$t_{true} = 12^{\mathrm{h}}\!:00^{\mathrm{m}} + 34^{\mathrm{m}} = 12^{\mathrm{h}}\!:34^{\mathrm{m}}. \tag{32}$$

The Calendar

There are a few recurring astronomical features that serve quite obviously as measurements of time. There is the daily rise of the sun that leads to the definition of a **day** and its divisions (hours, minutes and seconds). There are also the phases of the moon which are repeated (approximately) every 28 days leading to the notion of **month** and its subdivision, the **week**.

And then there is the time it takes the earth to complete one orbit around the sun which leads to the definition of **year** and to the recurrence of the seasons.

Unfortunately the number of days in a year or in a month is not an exact integer and this complicates the reckoning of the date. If one month were exactly 4 weeks (28 days), and if the year were exactly 12 months (336 days) there would be no difficulty. However, the year is closer to thirteen 28-day months. This leads to 364 days per year. The extra day could be declared a universal holiday. The trouble with this scheme is that it does not lend itself to an easy divisions in quarters. Thus, 12 months per year is the choice.

The first month of the Roman calendar used to be *Martius*, our present March, the fifth month of the year was *Quintilis*, the sixth, *Sextilis*, the seventh, *September*, and so on. From 153 B.C. on, *Ianuarius* was promoted to first place leading to September being the 9th month, not the 7th as before.

The exact date of the equinox could be easily measured (as was probably done at Stonehenge and at other much older observatories), and thus, one could easily establish that the Vernal Equinox should always occur at the same date (March, 21, for instance). This meant that occasionally one additional day would have to be added to the year. In this respect, the Romans were a bit sloppy and left the slippage accumulate until it became painfully obvious that the seasons were out of phase. If a given crop had to be planted at, say, the first day of spring, you could not assign a fixed date for this seeding day.

By the time the error became quite noticeable, the Pontifex maximus[†] would declare a *mensis intercalaris*, an intercalary month named *Mercedonius* and stick it somewhere toward the end of February, which was then at the end of the year. With time, the position of pontifex became a sinecure and the calendar adjustment became quite erratic and subject to political corruption. To correct things, by 46 B.C,, Julius Caesar declared a year 445 days long (three intercalary months were added). He also decreed a new calendar establishing that each year would be 365 days long, and, to account for the extra 1/4 day, every four years an additional day would be added. He commemorated this achievement by naming Quintilis after himself—thus introducing the name July. Not to be outdone, his nephew Octavian (Augustus) insisted in changing Sextilis to August. Of course, both July and August are 31-day months. What else?

The corrections worked for a while but not perfectly (because the year is not *exactly* $365\frac{1}{4}$ days). In March 1582, Gregory XIII introduced a fix—the Gregorian calendar used today. A number of European nations immediately adopted the new calendar; others resisted. England, always bound by

[†] Chief bridge builder, possible in charge of bridge maintenance in old Rome.

tradition, only adopted the Gregorian on September 2 1752. The next day became September 14 1752. So the answer to the trivia question "What important event occurred in the United States on September 10 1752?" is "Nothing!" By the way, Russia only converted to the Gregorian calendar on February 1 1918, and this is why the "October Revolution" actually occurred in November.

The Julian Day Number

In many astronomical calculation, it proves extremely convenient to have a calendar much simpler than any of those in common use. The simplest way to identify a given day is to use a continuous count, starting at some arbitrary origin in the past, totally ignoring the idea of year, month, and week. The astronomical Julian day number is such a system. We will define it as the day count starting with the number 2,400,000 corresponding to November 16, 1858. Thus, for instance, the next day, November 17, 1858, has a Julian day number of 2,400,001. The Julian day number starts at noon.

To determine the Julian day number corresponding to a given Gregorian date, it is sufficient to count the number of days after (or before) November 16, 1858 and add this to 2,400,000. Easier said than done. It is a pain to count the number of days between two dates. We suggest the following algorithm taken from *http://webexhibits. org/calendars/calendar-christian.html*:

$$JD = d + \mathrm{INT}((153m + 2)/5) + 365y + \mathrm{INT}(y/4) \\ - \mathrm{INT}(y/100) + \mathrm{INT}(y/400) - 32045.$$

In the above, y stands for the year (expressed in four figures), m is the month, and d is the day.

Notice that there is no year zero in the Julian or Gregorian calendars. The day that precedes January 1 1 A.D. is December 31 1 B.C. If you want a Julian day number of a B.C. date, to use the formula above, you must convert to negative year numbers, as, for instance, 10 B.C. must be entered as −9.

If you are dealing with ancient dates, you must realize that they are given, most frequently, in the Julian calendar, not the Gregorian, even if the are earlier than the year (45 BC) when the Julian calendar was established. This use of a calendar do express dates before it was established is an "anticipation" or "prolepsis" and is called a **proleptic** calendar. Do not confuse the Julian calendar with the Julian day number (different Juliuses, almost certainly).

For more information on Julian day numbers and on algorithms to convert Julian day numbers to Gregorian or to Julian dates, please refer to the URL given above.

Appendix B (Orbital Mechanics)

Sidereal versus Solar

The most obvious measure of time is the interval between two consecutive **culminations** of the sun, called a **solar day**. The sun culminates (reaches the highest elevation during a day—the smallest zenith angle) when it crosses the local meridian and is, consequently, exactly true south of an observer in the northern hemisphere. As will be explained later, there is an easily determined moment in the year when equinoxes occur. The time interval between two consecutive vernal equinoxes—that is, the length of the **tropical year** has been measured with—what else?— astronomical precision. It is found that during one tropical year, the sun culminates 365.24219878 times (call it 365.2422 times),[†]—in other words, there are, in this one-year interval, 365.2422 solar days. We define the **mean solar hour** as 1/24 of a mean solar day (a day that lasts 1/365.2422 years). Unfortunately, the length of a solar day as measured by any reasonably accurate clock changes throughout the year. The change is not trivial—the actual solar day (the time between successive culminations) is 23 seconds shorter than the mean solar day on September 17, and it is 28 seconds longer on December 22. These differences accumulate—in mid-February the sun culminates some 14 minutes after the mean solar noon and in mid-November, some 15 minutes before mean solar noon. We will explain later what causes this variability, which renders the interval between consecutive culminations an imprecise standard for measuring time.

If we were to measure the time interval between successive culminations of a given star, we would find that this time interval is quite constant—it does not vary throughout the year. We would also find that a given star will culminate 366.2422 times in a tropical year. The number of "star," or **sidereal**. days in a year is precisely one more day than the number of solar days.

The discrepancy between the sidereal and the solar time is the result of the earth orbiting *around* the sun. Refer to Figures 12.12 and 12.13.

In a planetary system in which a planet does not spin, its motion around the sun causes an observer to see the sun move *eastward* throughout the year resulting in one apparent day per year. If, however, the planet spins (in the same direction as its orbital motion) at a rate of exactly 360 degrees per year, then the sun does not seem to move at all—the planetary spin exactly cancels the orbitally created day.

[†] The length of a tropical year is decreasing very slightly, at a rate of $169 \times 10^{-10}\%$ per century. This means that there is a very small secular reduction in the orbital energy of earth.

Table 12.3
Time Definitions

Year (tropical)	Interval between two consecutive vernal equinoxes
Mean solar day	1/365.24219878 years
Mean solar hour	1/24 mean solar days
Minute	1/60 mean solar hours
Second	1/60 minutes
Sidereal day	1/366.24219878 years
Mean solar hours/year	8,765.81
Minutes per year	525,948.8
Seconds per year	31.5569×10^6
Length of sidereal day	23:56:04.09

As explained, the completion of a full orbit around the sun will be perceived from the surface of a nonspinning planet as one complete day—the sun will be seen as moving in a complete circle around the planet. If we count 24 hours/day, then each 15° of orbital motion cause an apparent hour to elapse, and 1° of orbital motion corresponds to 4 minutes of time.

Thus, in the case of the planet earth which spins 366.2422 times a year, the number of solar days is only 365.2422 per year because the orbital motion cancels one day per year.

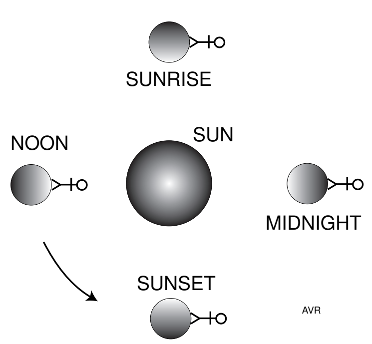

Figure 12.12 In a planetary system in which the planet does not spin, the orbital motion introduces one apparent day per year.

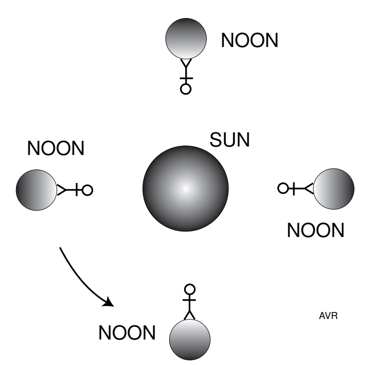

Figure 12.13 If the planet spins exactly once per year, the sun appears not to move.

Orbital Equation

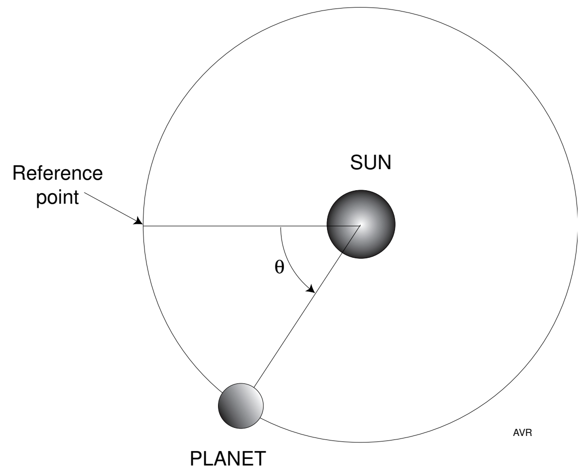

Figure 12.14 The angular position of a planet in its orbit in the ecliptic plane is called the true anomaly, θ.

The angle *reference point-sun-planet* is called the **true anomaly**. See Figure 12.14. Notice the quaint medieval terminology frequently used in astronomy.

In a circular orbit, there is no obvious choice for a reference point. However, since most orbits are elliptical (or, maybe, hyperbolic) the periapsis (nearest point to the attracting body)† is a natural choice. The apoapsis is not nearly as convenient because, in the case of long period comets, it cannot be observed.

Thus, the origin for the measurement of the true anomaly is the periapsis. The anomaly increases in the direction of motion of the planet.

Consider a planetary system consisting of a sun of mass, M, around which orbits a planet of mass, m. Assume that the orbital velocity of the planet is precisely that which causes the centrifugal force, $mr\omega^2$ (where ω is the angular velocity of the planet in radians/second) to equal the attracting force, GmM/r^2, where G is the gravitational constant (6.6729×10^{-11} $\text{m}^2\text{s}^{-2}\text{kg}^{-1}$). In this particular case, these two forces, being exactly of the same magnitude and acting in opposite direction, will perfectly cancel one another—the distance, r, from sun to planet will not change and the orbit is a circle with the sun at the center:

$$mr\omega^2 = G\frac{Mm}{r^2}, \tag{33}$$

† Here we have a superabundance of terms with nearly the same meaning: **periapsis, perihelion, perigee, periastron, pericenter,** and **perifocus** and, **apoapsis, aphelion, apogee, apastron, apocenter,** and **apofocus.**

from which

$$r = \frac{GM}{\omega^2}^{1/3}. \tag{34}$$

For the case of the sun whose mass is $M = 1.991 \times 10^{30}$ kg and the earth whose angular velocity is $2\pi/365.24$ radians per day or 199.1×10^{-9} radians per second, the value of r comes out at 149.6×10^9 m. This is almost precisely correct, even though we know that the orbit of earth is not circular.

In fact, it would be improbable that a planet had exactly the correct angular velocity to be in a circular orbit. More likely, its velocity may be, say, somewhat too small, so that the planet would fall toward the sun, thereby accelerating and reaching a velocity too big for a circular orbit and, thus, would fall away from the sun decelerating, and so on—it would be in an elliptical orbit.

The orbital equation can be easily derived, albeit with a little more math than in the circular case.

We note that the equations of motion (in polar coordinates) are

$$m\left[-\frac{d^2r}{dt^2} + r\left(\frac{d\theta}{dt}\right)^2\right] = G\frac{mM}{r^2} \tag{35}$$

and

$$m\left(2\frac{dr}{dt}\frac{d\theta}{dt} + r\frac{d^2\theta}{dt^2}\right) = 0 \tag{36}$$

for, respectively, the radial and the tangential components of force.

Equation 36 can be multiplied by r/m yielding

$$2r\frac{dr}{dt}\frac{d\theta}{dt} + r^2\frac{d^2\theta}{dt^2} = 0 \tag{37}$$

Note that Equation 35 becomes Equation 33 for the circular case (in which d^2r/dt^2 is zero). In this case Equation 36 reduces to $d^2\theta/dt^2 = 0$ (because dr/dt is also zero) showing, as is obvious, that the angular velocity is constant.

Note also that

$$\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) = 2r\frac{dr}{dt}\frac{d\theta}{dt} + r^2\frac{d^2\theta}{dt^2} = 0. \tag{38}$$

Integrating Equation 38,

$$r^2\frac{d\theta}{dt} = \text{constant}. \tag{39}$$

It is possible to show that Equations 35 and 36 represent, when t is eliminated between them, an equation of a conical section of eccentricity, ϵ (which depends on the total energy of the "planet"). When $\epsilon < 1$, the path or orbit is an ellipse described by

$$r = \frac{a(1-\epsilon^2)}{1+\epsilon\cos\theta} \qquad (40).$$

In the preceding, a is the semi major axis and θ is the true anomaly. Given the position, θ, of the planet in its orbit, it is easy to calculate the radius vector, provided the major axis and the eccentricity are know.

Of greater interest is the determination of the position, θ, as a function of time. From Equations 39 and 40,

$$\frac{d\theta}{dt} \propto \frac{(1+\epsilon\cos\theta}{a^2(1-\epsilon^2)^2} = A(1+\epsilon\cos\theta)^2. \qquad (41)$$

If the orbit of earth were circular, i.e., if $\epsilon = 0$, then the angular velocity, $d\theta/dt$ would be constant;

$$\frac{d\theta}{dt} = \frac{360^\circ}{365.2422 \text{ days}} = 0.98564733 \quad \text{degrees/day}. \qquad (42)$$

This is the mean angular velocity of earth. Thus,

$$A = 0.98564733 \quad \text{degrees/day}. \qquad (43)$$

and, because the eccentricity of earth is, at present, approximately 0.01670,

$$\frac{d\theta}{dt} = 0.98564733(1+0.0167\cos\theta)^2. \qquad (44)$$

Consequently, when $\theta = 0$—that is, at the moment of perihelion passage, the angular velocity of earth is 1.01884 degrees/day, while at aphelion the angular velocity is down to 0.95300 degrees/day.

A description of the motion of earth—that is, a table of θ as a function of time, can be obtained by evaluating

$$\theta_i = \theta_{i-1} + A(1+\epsilon\cos\theta_{i-1})^2\Delta t. \qquad (45)$$

θ_0 is made equal to zero, corresponding to the perihelion.

Good accuracy is achieved even when the time increment, Δt, is as large as 1 day.

It turns out that the perihelion is not a very useful point of reference. It is more convenient to use a more obvious direction, the vernal equinox, as initial point. For this, an **ecliptic longitude** is defined as the angle, measured along the ecliptic plane, *eastward* from the vernal equinox. The tabulation resulting from Equation 45 can be used provided the **longitude of the perihelion** also called the **argument of perihelion** is known. This quantity varies slowly but is, at present, close to $-77°$.

The heliocentric polar coordinate system used to derive the orbital motion of earth is not very convenient for describing the position of a celestial body as seen from earth. For this, the latitude/longitude system used in geography can easily be extended to astronomy. The various positions are described by a pair of angles: the **right ascension** equivalent to longitude and the **declination** equivalent to latitude. We are back to the old geocentric point of view although we recognize, as Aristarchus of Samos did back in around 200 B.C., that we are not the center of the universe. For such a system, the reference is the spin axis of the planet, a direction perpendicular to the **equatorial plane**, which passes through the center of earth. Only by extreme coincidence would the equatorial plane of a planet coincide with the **ecliptic** (the plane that contains the orbit of the planet). Usually, these two planes form an angle called **obliquity** or **tilt angle**, τ. It is possible to define a **celestial equator** as a plane parallel to the terrestrial equator but containing the center of the sun not that of the earth.

Celestial equator and ecliptic intersect in a line called the **equinoctial line**. When earth crosses this line coming from south to north (**the ascending node**), the **vernal equinox** occurs. At the descending node, when earth crosses the line coming from north, the **autumnal equinox** occurs.

The vernal equinox is used as a convenient origin for the measurement of both the ecliptic longitude and the right ascension. Remember that the former is an angle lying in the ecliptic plane while the latter lies in the equatorial plane.

The time interval between two consecutive vernal equinoxes is called the tropical year referred to at the beginning of this appendix. In our derivation, we used the perihelion as the origin for measuring the true anomaly. The time interval between two consecutive perihelion passages is called the **anomalistic year**, which, surprisingly, is slightly longer than the tropical year. How can that be? The reason for this discrepancy is that the line of apsides slowly changes its orientation completing 360° in (roughly) 21,000 years. The corresponding annual change in the longitude of the perihelion is $360/21,000 = 0.017$ degrees per year.

Since the orbital angular velocity of earth is roughly 1 degree per day, this means that the anomalistic year will be about 0.017 days (25 minutes) longer than the tropical one which is 365.242 days long. Hence, the anomalistic year should be about 365.259 days long (more precisely, 365.25964134 days).

Relationship Between Ecliptic and Equatorial Coordinates

Celestial longitude is measured along the ecliptic while right ascension —which is also a measure of longitude—is measured along the equatorial plane. Clearly, a simple relationship must exist between these coordinates.

Consider a right handed orthogonal coordinate system with the center of earth at the origin, in which the x-y plane coincides with the equatorial plane and the y-axis is aligned with the equinoctial line. The z-axis points north.

Let $\vec{s}$ be a unit vector, starting from the origin and pointing toward the sun,

$$\vec{s} = \vec{i}s_x + \vec{j}s_y + \vec{k}s_z. \tag{46}$$

This vector must lie in the ecliptic plane—that is, it must be perpendicular to the spin axis whose unit vector is

$$\vec{u} = -\vec{i}\sin\tau + \vec{k}\cos\tau. \tag{47}$$

If $\vec{s}$ is perpendicular to $\vec{u}$, then their dot product must be zero:

$$\vec{s}\cdot\vec{u} = 0 = -s_x \sin\tau + s_z \cos\tau, \tag{48}$$

$$s_z = s_x \tan\tau \tag{49}$$

The reference for measuring longitude and right ascension is the direction of the vernal equinox, which, in our co-ordinate system has a unit vector $-\vec{j}$. Hence, the longitude, Λ, is given by

$$\cos\Lambda = -s_y. \tag{50}$$

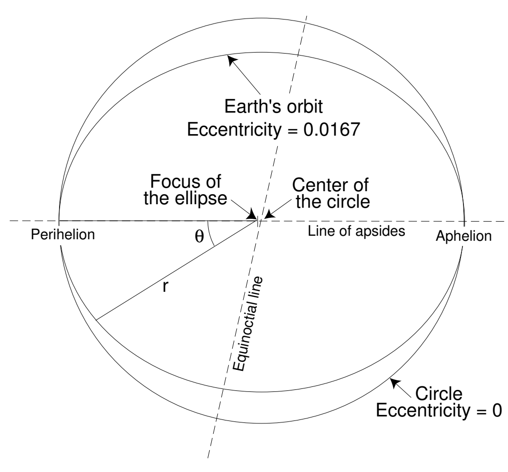

Figure 12.15 The orbit of earth is slightly elliptical. The view is normal to the ecliptic.

Since $\vec{s}$ is a unit vector,

$$s_x^2 + s_y^2 + s_z^2 = s_x^2 + \cos^2 \Lambda + s_x^2 \tan^2 \tau = 1, \tag{51}$$

$$s_x^2(1 + \tan^2 \tau) = 1 - \cos^2 \Lambda, \tag{52}$$

$$s_x = \sin \Lambda \cos \tau. \tag{53}$$

But on the equatorial plane, the right ascension, $\Re$, is given by

$$\tan \Re = \frac{s_x}{-s_y} = \frac{\sin \Lambda \cos \tau}{\cos \Lambda} = \cos \tau \tan \Lambda. \tag{54}$$

$$\Re = \arctan(\cos \tau \tan \Lambda). \tag{55}$$

As usually, when one takes reverse trigonometric functions, the answer is ambiguous—the calculator or the computer gives only the principal value and a decision must be made of what the actual value is. In the present case, the following Boolean statement must be used:

$$\begin{gathered}\text{If } \Lambda >= 90^\circ \text{ AND } \Lambda < 270^\circ \text{ THEN } \Re = \Re + 180^\circ \\ \text{ELSE IF } \Lambda >= 270^\circ \text{ THEN } \Re = \Re + 360^\circ.\end{gathered} \tag{56}$$

This relates the longitude of the sun to its right ascension.

The declination of the sun is,

$$\sin \delta = s_z = s_x \tan \tau = \sin \Lambda \cos \tau \tan \tau = \sin \Lambda \sin \tau. \tag{57}$$

The Equation of Time

We have now to explain why the time between successive solar culminations varies throughout the year:

Assume you have on hand two instruments:

1. An accurate clock that measures the uniform flow of time calibrated in mean solar time. From one vernal equinox to the next, it registers the passage of $365.2422 \times 24 \times 60 \times 60 = 31.55692 \times 10^6$ seconds.
2. A sundial that can measure the solar time with a resolution of at least one minute.†

Set your clock to start at exactly noon (as seen in the sundial) on any arbitrary date. It will be noted that, although by the same date of the next year, the clock and the sundial are again synchronized, throughout

† Owing to the finite angular diameter of the sun (0.5°), it is difficult to read a sundial to greater precision than about 1 minute of time.

the year, the sundial seems sometimes to be slow and, at other periods of the year, to be fast. The difference between the sundial time and the clock time—between the solar time and the mean solar time—may reach values of up to 15 minutes fast and 15 minutes slow. This difference is called the **Equation of Time, EOT**.

Figure 12.19 shows how the EOT varies along the year. For planning solar collectors, it is sufficient to read the value of the EOT off the figure. For such an application, the empirical formula below (Equation 59) is an overkill.† It yields the EOT in minutes when the day of the year, d, is expressed as an angle, d_{deg}:

$$d_{deg} \equiv \frac{360}{365} d \quad \text{degrees.} \tag{58}$$

Greater precision may be useless because the EOT varies somewhat from year to year with a 4 year period, owing to the leap years.

$$\begin{aligned} EOT = & -0.017188 - 0.42811 \cos(d_{deg}) + 7.35141 \sin(d_{deg}) \\ & + 3.34946 \cos(2 d_{deg}) + 9.36177 \sin(2 d_{deg}) \quad \text{minutes} \end{aligned} \tag{59}$$

We recall that there are two measures of solar longitude, both increasing eastward from the direction of the vernal equinox: One measure is along the ecliptic and is called the ecliptic longitude, Λ, the other is measured along the equatorial plane and is called the right ascension, $\Re$. These two longitudes are exactly the same at the equinoxes and at the solstices, but are different anywhere in between. See Table 12.4.

Table 12.4
Difference Between Longitude and Right Ascension

Season	$\Lambda - \Re$
Spring	> 0
Summer	< 0
Autumn	> 0
Winter	< 0

† The formula is from a NOAA program: http://www.srrb.noaa.gov/highlights/sunrise/program.txt.

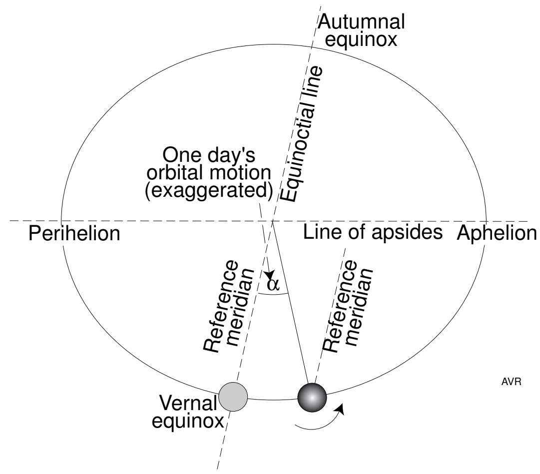

Figure 12.16 As the earth moves in its orbit, the sun appears to move eastward.

The discrepancy between Λ and $\Re$ increases with the obliquity of the orbit. If the obliquity of earth's orbit were zero, then $\Lambda = \Re$ under all circumstance.

Assume that you are on the surface of earth at an arbitrary latitude but on a meridian that happens to be the one at which the sun culminates at the exact moment of the vernal equinox. At this moment $\Lambda = \Re = 0$.

23^{h}: 56^{m}: $04^{\mathrm{s}}.09$ or 23.93447 h later, earth has completed a full rotation and your meridian faces the same direction it did initially.

However, the sun is not exactly at culmination. The reason is that the orbital motion of earth has caused an apparent eastward motion of the sun. The earth has to spin another α degrees for the reference meridian to be facing the sun again. See Figure 12.16.

If the orbit were circular and the obliquity zero, then the uniform eastward motion of the sun would be $360°/365.2422 = 0.985647$ degrees/mean solar day. In more technical terms, the rate of change of the anomaly would be constant and the **mean anomaly** would be

$$< \theta >= 0.985647t, \tag{60}$$

where t is the time (in mean solar days) since perihelion passage. The ecliptic longitude changes at the same rate as the anomaly. Since the spin rate of earth is 360° in 23.93447 hours or 15.04107°/hour, or 0.2506845° per minute, it takes $0.985647/0.2506845 = 3.931823$ minutes (0.06553 hours) to rotate the 0.985647° needed to bring the reference meridian once again under the sun—that is, to reach the next culmination or next noon. Not surprisingly, $23.93447 + 0.06553 = 24.00000$ hours. This means that in this simple case, consecutive noons are evenly spaced throughout the year and occur exactly 24 hours apart, a result of the definition of the solar mean hour.

In reality noons do not recur at uniform intervals—that is, the sun dial time does not track exactly the clock time. Twenty-four hours between noons is only the value averaged over one year. The difference between the clock time and the sun dial time is called, as stated before, the equation of time, EOT. Two factors cause this irregularity: the eccentricity of the orbit (leading to $EOT_{eccentricity}$) and the obliquity of the orbit (leading to $EOT_{obliquity}$).

Orbital Eccentricity

If the orbit is not circular, the rate of solar eastward drift, (the rate of change of the anomaly) is not constant. It changes rapidly near perihelion and more slowly near aphelion. Hence, the true anomaly differs from the mean anomaly so that, after each 24 mean solar hours period, the earth has to spin an additional $\theta - <\theta> \equiv C$ degrees. Here the difference, C, between the true anomaly and the mean anomaly is called the **equation of center**, another example of medieval terminology.

Since the spin rate of earth is very nearly 1 degree in 4 minutes of time, the time offset between the true noon and the mean noon owing to the eccentricity of the orbit is

$$EOT_{eccentricity} = 4C \quad \text{minutes of time.} \tag{61}$$

In the preceding, the angle, C, is in degrees.

The eccentricity component of the equation of time varies throughout the year in a sinusoidal fashion with zeros at the perihelion and aphelion and with extrema of 8 minutes of time midway between these dates. Figure 12.17 shows these variations.

The value of C for any time of the year can be found by calculating θ using Equation 44 and subtracting the mean anomaly obtained from Equation 60. For many applications, it may prove more practical to calculate C using the empirical equations below:[†]

$$<\theta> = 357.52911 + 35999.05029T - 0.0001537T^2 \tag{62}$$

$$\begin{aligned} C = {} & (1.914602 - 0.004817T - 0.000014T^2)\sin <\theta> \\ & + (0.019993 - 0.000101T)\sin(2 <\theta>) \\ & + 0.000289\sin(3 <\theta>) \quad \text{degrees.} \end{aligned} \tag{63}$$

Here, T, is the number of Julian centuries since January 1 2000. See farther on for explanation of Julian dates.

[†] The formulas are from a NOAA program:
http://www.srrb.noaa.gov/highlights/sunrise/program.txt.

Orbital Obliquity

If the orbit is circular but the obliquity is not zero, then although the rate of solar ecliptic longitude increase (or the rate of the anomaly increase) is constant, the rate of change of the right ascension is not. The moment of solar culmination is related to the right ascension.

On the day after the vernal equinox, as seen by an observer on the reference meridian on earth, the sun is at a right ascension of

$$\begin{aligned}\Re &= \arctan(\cos\tau\tan\Lambda) = \arctan\big(\cos(23.44°)\tan(0.985647°)\big)\\ &= \arctan(0.91747\times 0.017204) = \arctan\big(0.015785\big),\end{aligned} \tag{64}$$

$$\Re = 0.904322° \tag{65}$$

For the sun to culminate, the earth has to spin an additional 0.904322° rather than 0.985647° as in the case of zero obliquity. Thus, noon will occur somewhat earlier than in the zero obliquity situation. In fact it will occur 4×(0.985647-0.904322)=0.325 minutes earlier.

Generalizing,

$$EOT_{obliquity} = 4(\Lambda - \Re). \tag{66}$$

The obliquity component varies, as does the eccentricity one, in a sinusoidal fashion but completes two cycle rather than one in the space of one year. The zeros occur at the equinoxes and solstices instead at the perihelion and aphelion as it does in the eccentricity case. The amplitude of $EOT_{obliquity}$ is 10 minutes. This behavior is depicted in Figure 12.18, while the behavior of the full EOT (the sum of the eccentricity and the obliquity components) is depicted in Figure 12.19.

It is important not to confuse perihelion (when the earth is closest to the sun) or aphelion (when it is farthest) with the solstices which occur when the solar declination is an extremum—that is, when $\delta \pm 23.44°$. As it happens, the dates of the solstices are near those of perihelion and aphelion, but this is a mere coincidence. There are roughly 12 days between the (summer) solstice and the aphelion and between the (winter) solstice and the perihelion. See Table 12.5.

Table 12.5
Dates of Different Sun Positions

	Approx. Date	Approx Day No.
Perihelion	2...4 January	2
Vernal Equinox	20...21 March	80
Solstice (Summer, N. Hemisphere)	21 June	172
Aphelion	4...5 July	184
Autumnal Equinox	22...23 September	262
Solstice (Winter, N. Hemisphere)	21...22 December	355

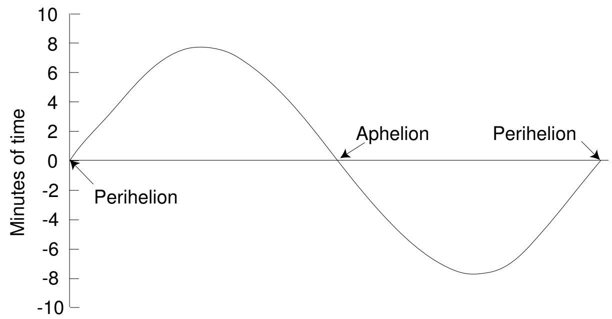

Figure 12.17 That part of the equation of time resulting from the ellipticity of earth's orbit has a value (in minutes of time) equal to $4C$, where C is the equation of center—that is, the difference between the true anomaly and the mean anomaly. Observe that this function has a zero at both perihelion and aphelion.

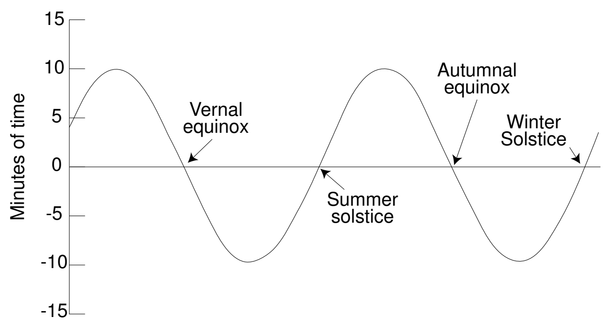

Figure 12.18 The part of the equation of time resulting from the obliquity of earth's orbit has a value (in minutes of time) equal to $4(\Lambda - \Re)$—that is, 4 times the difference between the solar longitude and its right ascension. Zeros occur at the equinoxes and solstices.

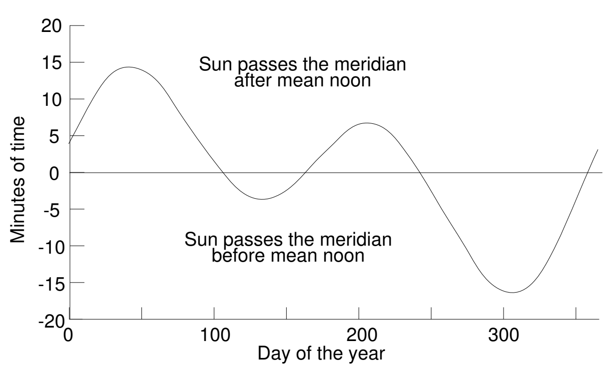

Figure 12.19 The observed equation of time is the combination of the effects owing to ellipticity and obliquity of earth's orbit.

12.38

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PROBLEMS

12.1 A time traveler finds himself in an unknown place on Earth at an unknown time of the year. Because at night the sky is always cloudy, he cannot see the stars, but he can accurately determine the sunrise time and also the length of a shadow at noon. Sun rises at 0520 local time. At noon, a vertical mast casts a shadow 1.5 times longer than its height. What is the date and what is the latitude of the place? Is this determination unambiguous?

12.2 An astronaut had to make an emergency de-orbit and landed on an island in the middle of the ocean. He is completely lost but has an accurate digital watch and a copy of "Fundamentals of Renewable Energy Processes" that NASA always supplies†. He times carefully the length of the day and discovers that the time between sunrise and sunset is 10:49:12. He knows the date is Jan. 1 1997. He can now figure his latitude. Can you?

12.3 A building in Palo Alto, CA (latitude 37.4 N) has windows facing SSE. During what period of the year does direct light from the sun enter the window at sunrise? Assume no obstructions, good weather and a vanishing solar diameter.

What is the time of sunrise on the first day of the period? And on the last day? What is the insolation on the SSE-facing wall at noon at the equinoxes?

12.4 Consider an ideal focusing concentrator. Increasing the concentration causes the receiver temperature to increase—up to a point.

Beyond certain maximum concentration, the temperature remains constant. What is the maximum concentration that can be used on Mars, for a 2-D and for a 3-D case?Some data:

Radius of the orbit of Mars is 1.6 AU.
1 AU is 150 million km.
The angular diameter of the moon, as seen from Earth, is 0.5 degrees.

12.5 Consider the arbitrary distribution function

$$\frac{dP}{df} = f - \frac{1}{2}f^2$$

Determine for what value of f is this function a maximum.
Plot $\frac{dP}{df}$ as a function of f for the interval in which $\frac{dP}{df} > 0$.
Now define a new variable, $\lambda \equiv c/f$, where c is any constant.
Determine for what value of f is the distribution function $\frac{dP}{d\lambda}$ a maximum.
Plot $|\frac{dP}{d\lambda}|$ as a function of f.

12.6 An expedition to Mars is being planned. Let us make a preliminary estimate of the energy requirements for the first days after the expedition lands.

Landing date is November 15, 2007 which is Mars day 118. At the landing site (17.00° N, 122° E) it will be just after local sunrise. The five-person landing team has all of the remaining daylight hours to set up the equipment to survive the cold night.

Prior to the manned landing, robots will have set-up a water extraction plant that removes the liquid from hydrated rocks by exposing them to concentrated sunlight. Assume an adequate (but not generous supply of water).

Power will be generated by photovoltaics and will be stored in the form of hydrogen and oxygen obtained from water electrolysis.

The photovoltaics are blankets of flexible material with 16.5% efficiency at 1 (Mars) sun. No concentrators will be used. These blankets will be laid horizontally on the Mars surface.

The electrolyzers operate at 90% efficiency.

MARS DATA (relative to Earth)

Mean radius of orbit	1.52
Gravity	0.38
Planetary radius	0.53
Length of day	1.029
Length of year	1.881
Density	0.719

The inclination of the plane of the Martian equator referred to the plane of its orbit is 25.20°.

The inclination of the plane of the Martian orbit referred to the ecliptic is 1.85°.

The average daytime Martian temperature is 300 K (just a little *higher* than the average daytime Earth temperature of 295 K). The Martian night, however, is cold! Average temperature is 170 K (versus 275 K, for Earth).

The vernal equinox occurs on Mars day 213.†

Define a Martian hour, h_m as 1/24 of the annual average of the period between consecutive sunrises.

1. How long does the sunlit period last on the day of arrival?
2. Determine the available insolation on a horizontal surface (watts m^{-2}, averaged over a Martian day—that is, over a 24 h_m period).

† Not really. Although the other data are accurate, this date was pulled out of the hat.

3. Estimate the O_2 consumption of the five astronauts. Consider that they are on a strictly regulated diet of 2500 kilocalories per Martian day. Assume that all this is metabolized as glucose. Use 16 MJ per kg of glucose as combustion enthalpy.
4. How much energy will be required to produce the necessary amount of oxygen from water by electrolysis?
5. What area of solar cell blanket must be dedicated for the production of oxygen?
6. Assume that the mean temperatures of Mars are the actual temperatures at the astronauts settlement. Assume further that the temperature of the Martian air falls instantaneously from its daytime 300 K to its nighttime 175 K and vice-versa.

   The astronauts are housed in an hemispherical plastic bubble 10 m in diameter. The wall material is rated at R-12 (in the American system) as far as its thermal insulation is concerned. No heat is lost through the floor.

   The interior of the bubble is kept at a constant 300 K. During the night, stored hydrogen has to be burned to provide heat. The inside wall of the bubble is at 300 K, while the outside is at 175 K. Assume that the effective emissivity of the outside surface is 0.5.

How much hydrogen is need per day? Express this in solar cell blanket area.

12.7 How long was the shadow of a 10-m tall tree in Palo Alto, CA (37.4 N, 125 W) on March 20 1991 at 0200 PM (PST)? Desired accuracy is ± 20 cm.

12.8 You are on a wind swept plain on the planet Earth but you know neither your position nor the date. There are no hills and you can see the horizon clearly. This allows you to time the sunset accurately, but unfortunately your watch reads Greenwich time. Take a straight pole and plant it exactly vertically in the ground (use a plumb line). You have no meter- or yardstick but you assign arbitrarily 1 unit to the length of the pole above ground. Observe the shadow throughout the day: at its shortest (at 08:57 on your watch) the shadow is 2.50 units long. Sun sets at 13:53. From these data alone, determine the date, latitude and longitude unambiguously (i.e., decide if you are in Northern or the Southern hemisphere).

12.9 Calculate the azimuth of a vertical surface that results in the maximum annual average relative insolation under the conditions below:

The surface is situated at latitude of 40° N in a region in which there is always a dense early morning fog (insolation = zero) up to 10:00 and then the rest of the day is perfectly clear.

Relative insolation is defined here as the ratio of the insolation on the surface in question to that on a *horizontal* surface at the equator, on the same day of the year.

12.10 At what time of day is the sun due east, in Palo Alto?

The latitude of Palo Alto is 37.4° north; the longitude is 122° west of Greenwich.

Do this for two days of the year: August 11 and November 15.

12.11 What is the insolation (W m^{-2}) on a surface facing true east and with 25° elevation erected at a point 45° N at 10:00 local time on April 1 1990?

Assume that the insolation on a surface facing the sun is 1000 W m^{-2}.

12.12 The average food intake of an adult human is, say, 2000 kilocalories per day. Assume that all this energy is transformed into a constant uniform heat output.

Ten adults are confined to a room $5\times5\times2$ meters. The room is windowless and totally insulated with R-11 fiberglass blankets (walls, door, ceiling, and floor). Outside the room the temperature is uniformly at 0 C. The air inside the room is not renewed. Assume that temperature steady state is achieved before the prisoners suffocate. What is the room temperature?

12.13 Here is a way in which a person lost in a desert island can determine her/his latitude with fair precision even though the nights are always cloudy (Polaris cannot be seen).

Use a vertical stick and observe the shadow. It will be shortest at local noon. From day to day, the shortest noon shadow will change in length. It will be shortest on the day of the summer solstice: this will tell you the solar declination, δ.

On any day, comparing the length of the horizontal shadow with the length of the stick, will let you estimate the solar zenithal angle, χ, quite accurately, even if no yard- (or meter-) stick is on hand.

Knowing both δ and χ it is possible to determine the latitude. Or is it?

Develop a simple expression that gives you the latitude as a function of the solar declination and the noon zenithal angle. No trigonometric functions, please! Is there any ambiguity?

12.14 As determined by a civilian-type Trimble GPS receiver, the latitude of my house in Palo Alto, CA is 34.44623° N.

At what time of the year are consecutive sunrises exactly 24 hours apart?

Call Δt the time difference between consecutive sunrises. At what time of the year is Δt a maximum?

What is the value of this maximum Δt? Express this in seconds.

12.15 An explorer in the Arctic needs to construct a cache in which to store materials than cannot stand temperatures lower than −10 C. The surface air over the ice is known to stay at −50 C for long periods of time.

The cache will be built by digging a hole in the ice until the bottom is 0.5 meters from the water. The roof of the cache is a flat surface flush with the ice surface. It will be insulated with a double fiber glass blanket, each layer rated at R-16 (in the American system).

The heat conductivity of ice is 1.3 W $\text{m}^{-1}\text{K}^{-1}$.

Assume that the area of the cache is so large that the heat exchange through the vertical walls can be neglected.

Estimate the temperature inside the cache when the outside temperature has been a steady -50 C for a time sufficiently long for the system to have reached steady state. Assume the temperature inside the cache is uniform.

12.16 What is the solar azimuth at sunset on the day of the summer solstice at 58° north?

12.17 A battery of silicon photocells mounted on a plane surface operates with an efficiency of 16.7% under all conditions encountered in this problem. It is installed at a location 45° north. The time is 10:00 on April 1 1995.

When the battery faces the sun directly, it delivers 870 W to a resistive load. How much power does it deliver if the surface is set at an elevation of 25° and faces true east?

12.18 Consider a mechanical heat pump whose coefficient of performance is exactly half of the ideal one. It uses an amount W of mechanical energy to pump Q_C units of energy from an environment at −10 C into a room at 25 C where an amount $Q_A = Q_C + W$ of energy is deposited. How many joules of heat are delivered to the room for each joule of mechanical energy used?

12.19 You have landed on an unknown planet and, for some obscure reason, you must determine both your latitude and the angle, ι, the inclination of the planet's spin axis referred to the its orbital plane.

To accomplish such a determination, all you have is a ruler and plenty of time. Erect a vertical pole, exactly 5 m tall and observe the length of the shadow at noon as it varies throughout the year.

The shortest length is 1.34 m and the longest is 18.66 m.

12.20 The smallest zenithal angle of the sun was, on January 1 2000, 32.3°. At that moment, the sun was to your south. What is your latitude?

12.21 You are 30° north. The day is September 15 of a nonleap year. It is 12:44 true solar time. A 10 meter long rod tilted due west is planted in the ground making a 30° angle with the vertical.

Calculate the position of the sun.

What is the length of the shadow of the rod?

12.22 On which day of the year does the sun appear vertically overhead at the three locations below. Determine the time (in standard time) in which this phenomenon occurs. Disregard the Equation of Time. Also assume that the time zones are those dictated by purely geographic considerations, not by political ones.

Locations:

Palo Alto, California (USA): $37^\circ\ 29'$ N, $122^\circ\ 10'$ W.

Macapá, Amapá (Brazil): 0.00, $51^\circ\ 07'$ W.

Brasília, Federal District (Brazil): $15^\circ\ 53'$ S, $47^\circ\ 51'$ W.

12.23 If you consult the URL <http://mach.usno.navy.mil/> and follow the instructions, you will find that for San Francisco, CA (W122.4, N37.8) on February 19 2002, the sunrise occurred at 06:55 and the sun crossed the meridian at 12:24.

1. Using the information in the textbook, verify the meridian crossing time.
2. Still using the formulas in the textbook, calculate the sunrise time. If there is any discrepancy, indicate what causes it.

12.24 Aurinko (pronounced OW-rin-ko, where the stressed syllable, OW, has the same sound as in "how") is a (fictitious) unmanned airplane designed to serve as a repeater for radio signals replacing expensive satellites. It is equipped with 14 electric motors, each of which can deliver 1.5 kW. Its cruising speed is 40 km/hr (slightly faster than a man can run in a 100- or 200-meter dash). It operates at 30 km altitude.

Wingspan: 75.3 m.

62,120 solar cells.

Maximum electric output of the cells: 32 kW when full sunlight is normal to the cells.

1. When Aurinko is in its orbit (30 km above sea level) how far is its geometric horizon?† The geometric horizon differs from the radio horizon because the latter is somewhat extended by atmospheric refraction.
2. What is the area on the ground that is reached by the direct rays from the satellite. Disregard atmospheric refraction.
3. Assume that the airplane orbits a point 37.8° north. On the day in which the sunlight hours are the least, how long is the sunlit period as seen from the airplane at 30 km altitude? Disregard atmospheric refraction.
4. Assuming that the solar cells mounted on top of the wing of the airplane are always horizontal, what is the insolation averaged over the sunlit period on the day of the previous question? Since the airplane is above most of the atmosphere, the solar constant can be taken as 1200 W/m^2.
5. Assume that the total power consumption of the airplane while in orbit is 10 kW. (This includes propulsion, house keeping and communications.) The electric energy obtained from the photovoltaic array is in part directly used by the load and the excess is stored to be used during the period when the array output is less than the load demand. The storage system has a turn-around efficiency of $\eta_{turnaround}$—that is, only a fraction, $\eta_{turnaround}$, of the energy fed into the system can be retrieved later.

Assume, for simplicity, that $\eta_{turnaround} = 1.0$. Assume also that the efficiency of the photovoltaic collectors is 20% independently of the sun power density.

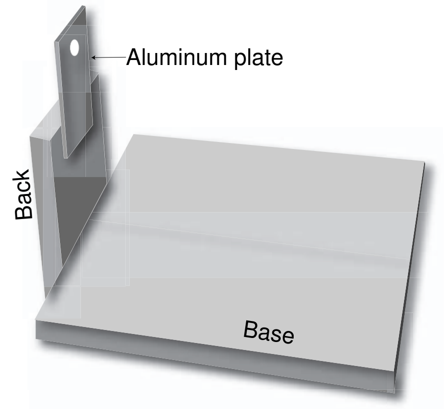
The solar array covers all the wing surface except for a rim of 20 cm. More clearly, there is a space of 20 cm between the leading edge and the array and the same space at the trailing edge and the wing tips. Consider rectangular (nontapered wings).

6. What must the chord of the wing be (chord = distance between leading and trailing edges)?

12.25 In the solar spectrum, what fraction of the total power density is absorbed by silicon? Hint: Use Table 12.1.

12.26 This is an experiment technically easy to carry out. Unfortunately, it takes 365 days to complete. Hence, we are going to invert the procedure and, from the theory developed in the textbook, we will calculate what the results of the experiment would be. If you have not been exposed to it, you may be surprise with what you get.

† This is not a space orbit around the planet. It is and orbit (as the term is used in aviation) around a point 30 km above sea level.



To set up the experiment, you would have to build the simple device illustrated in the figure. It consists of a wooden base large enough to support a standard piece of paper. A vertical piece of wood ("back") is attached to the base and mounted on it is a thin aluminum rectangle with a small hole in it. The aluminum must be thin enough (say, 1 mm or less) so that the sun can shine through the hole even when it is far from perpendicular to the plate.

In the model we used, the hole was 129 mm above the base, but this is not a critical dimension. Orient the device so that the hole faces equatorward. The noon sun will cast a shadow, but shining through the hole will cause a little dot of light to appear on the base. What we want to do is to follow the path this dot of light will traces out. The exact time in which the observations are made is important. You have to start at the astronomical noon on one of the following days (in which the equation of time, EOT, is zero): April 15, June 14, September 2, or December 25.

To follow the dot of light, place a sheet of paper on the base. Immobilize it by using some adhesive tape. With a pen, mark the center of the dot of light at the moment the sun culminates. Since on the suggested starting dates the equation of time is zero, the sun will culminate (i.e., cross the local meridian) at 1200 Standard Time corrected by the longitude displacement. This amounts to 4 minutes of time for each degree of longitude away from the longitude of the meridian at the center of the time zone. The center of the time zone is (usually) at exact multiples of 15° of longitude. For instance, the Pacific Time Zone in North America is centered on 120° W. If you happen to be in Palo Alto (125° W, your time offset (on days when the EOT is zero) is $15° \times (120° - 125°) \times 4 = -20$ minutes. Thus, the sun will culminate at 1220 PST.

The next observation must be made 240 hours (or an exact multiple of 24 hours) late—that is, it must, if you are in Palo Alto, be made exactly t 1220 PST. After one year, an interesting pattern will emerge.

12.47

The assignment in this problem is to calculate and plot the position of the light dot every 10 days over a complete year. Start on April 15 and do not forget the equation of time. Assume that you are in Palo Alto, CA: 125° W 37.4° N.

12.27 The time interval between two consecutive full moons is called **a lunar month**, or a **lunation**, and lasts, on average 29.53 days. Explain why this is not the **length of the lunar orbital period**—that is, the time it takes for the moon to complete one full orbit around the earth.

Calculate the length of the lunar orbital period.

12.28 A supraluminar (faster than light) probe was sent to reconnoiter an earth-like extra-solar planet whose characteristics are summarized in the table below. All units used are terrestrial units, except as noted below. It was determined that the planet had a very tenuous atmosphere, totally transparent to the solar radiation. As the probe materialized in the neighborhood of the alien sun, it measured the light power density which was 11 kW/m^2. The measurement was made when the probe was at exactly 50 million km from the sun. It was also determined that the planet was in an essentially circular orbit.

"Day" is defined as the time period between consecutive solar culminations (noons). It differs from the terrestrial day.

"Year" is the length of the orbital period—the time period between successive vernal equinoxes. It differs from the terrestrial year.

The "date" is designed by the day number counting from Day Number 1—the day of the vernal equinox.

Orbital radius	132×10^6	km
Orbital period	28.98×10^6	sec
Orbital inclination	26.7	degrees
Planetary radius	5,800	km
Spin period	90,000	sec

The direction of the spin is the same as the direction of the orbital motion.

The probe will land at at point with a latitude of +45°.

1. At the probe landing site, what is the length, in seconds, of the longest and of the shortest day-light period of the year?
2. What are the dates of the solstices?
3. What is the daily average insolation on a horizontal surface on day 50?